Chapter 1

Early Calculation

Introduction

This chapter covers many different aspects of the history of calculation, describing the first steps in numeration and continuing through some of the nineteenth- and twentieth-century developments of mechanical calculating machinery. It is quite impossible to make this story completely chronological because of many different overlapping developments; however, an effort has been made to show the broad flow of historical events in the approximate order in which they occurred. Some topics, for example the contributions of the nineteenth-century British mathematician Charles Babbage, are left to be described in other chapters because they logically belong to a different line of development than that described here.

The main emphasis in this chapter is on the historical development of mechanical aids to calculation. By the early 1600s the progress of calculation takes two different routes: the first is based on the mathematical development of logarithms and leads into a discussion of John Napier, Napier's bones, logarithms, and slide rules; the second is more of a mechanical than an intellectual achievement and leads into the early development of calculating machinery, finally culminating with the very sophisticated desk-top machines of the early twentieth century.

Numeration

Counting

We will never know when or how humans first developed the ability to count. The process does not leave any physical evidence behind for archaeologists to find. What we do know is that the process is extremely ancient. Any of the so-called primitive peoples that have been studied have all had a highly developed sense of number and, to at least some degree, an ability to represent numbers in both words and symbols.

Of course the very earliest civilizations would not have had the same need for a sophisticated number system, or the arithmetic that goes with it, as we do today. In fact, the general level of numerical knowledge that we now take for granted is a fairly recent development for the common individual. Some evidence of this can be found in that, prior to the eleventh century, British law stated in order for a man to be considered as a creditable witness in court, he had to be able to count up to nine. To apply such a criterion today would be ridiculous.

Once humans had developed the ability to count, it must have become necessary to have a method of recording numbers. Elementary situations do not require any sophisticated numeral system, just an ability to reconstruct the final figure at some later date. A typical instance would be the shepherd who puts one pebble in a bag for every sheep he lets out of the pen in the morning and removes one for every sheep herded back at night. If pebbles are left over after all the sheep are back in the pen, he knows that he has to go back and look for the strays.

Written Number Systems and Arithmetic

Humanity's first attempt at numerical notation was likely a simple pictorial system in which five cows would be drawn to represent five cattle or, with a slight generalization, seven tents might represent seven family groups. This pictorial stage is of very little interest from the point of view of the development of any arithmetic abilities, which did not usually arise until various civilizations had developed reasonably sophisticated systems of numerical notation.

The physical evidence we have, at the moment, seems to indicate that several different groups in different parts of the world had reached this stage by about 3000 B.C.

Once a culture had reached the point at which semipermanent recording of numerical information was necessary, the actual system that they developed appears to have been dependent on such factors as the type of writing materials available, the base of the number system being used, and cultural factors within the group. These cultural factors eventually dictated which of the two major notational systems, the additive or the positional, was adopted.

The additive notational system uses one distinct symbol to represent each different unit in the number base, this symbol being repeated as often as necessary to indicate the magnitude of the number being written. The classic example of an additive system is the one developed by the ancient Egyptians; however, for purposes of illustration, the Old Roman Numeral system will be much more familiar.

The Modern Roman Numeral system, which uses the subtractive forms of IV for 4 and IX for 9, is a development out of the Old Roman Numeral system, which, although it was seen as early as A.D. 130, did not become popular until about A.D. 1600. In the Old Roman system it was possible to express any number less than 5,000 by a sequence of symbols in which no individual sign needed to be repeated more than four times. For example, the number 3,745 would be represented as MMMDCCXXXXV. It was the custom to write down the symbols in decreasing order of their magnitude (M = 1000, D = 500, C = 100, X = 10, V = 5, I = 1), but this was not necessary. The same number could have been represented as CXXCXXMMVMD, but it never was because of the obvious ease of reading the number when the symbols are written in the order of descending value.

The pure additive system of notation is quite easy to use for simple calculations, even though it does not appear so at first glance. Addition involves the two step process of simply writing down the individual symbols from each number, then collecting together the sequences of smaller valued symbols to make larger valued ones so that the number regains its canonical form. For example:

2319	=	MM CCC X V IIII
+ 821	=	DCCC XX I
3140	=	MMDCCCCCCXXXVIIIII

The second step now takes over and, because IIIII = V, VV = X, CCCCC = D, and DD = M, the final result is written as MMMCXXXX.

Multiplication, although slow, is not really difficult and only involves remembering multiples of 5 and 10. For example:

28	=	XXVIII
x 12	=	XII
336		

XXVIII x I	=	XXVIII
XXVIII x I	=	XXVIII
XXVIII x X	=	CCLXXX
		CCLXXXXXXXVVIIIIII

which would be written as CCCXXVI.

The operations of division and subtraction are a little more cumbersome; however, they were aided by standard doubling and halving operations (as was multiplication) which are no longer in use today. These techniques of "duplation" and "mediation" were actually developed from similar methods used by the Egyptians.

Although more cumbersome than systems of positional notation, the additive systems are not without their merits, and computation is not difficult once the rules are mastered. The modification of such a number system to include subtractive elements, such as the IV = 4 or IX = 9 of the Modern Roman system, tend to make matters very much more difficult as far as arithmetic is concerned, but this device is not to be found at all in most examples of additive notation.

In positional number systems, like the one most of us use today, the values being represented are denoted entirely by the position of the symbol in the string of characters representing the number. Each position corresponds to a certain power of the 'base' being used. The base in most common use today is, of course, ten; the positions representing units, tens, hundreds, thousands, etc. This means that it is necessary to have a zero symbol to indicate an empty position. The Chinese actually had a mechanism of using a positional number system without a zero symbol, but this is very much the exception in this type of notation.

The rules of calculation in a positional system are more complex than those used with additive systems, and they usually require that the user memorize some form of multiplication table. Because of the fact that everyone is familiar with the working of our own positional number system, no attempt will be made to describe it in detail.

The Abacus

Introduction

The abacus is usually considered as being an object in the same class as a child's toy. This is quite the wrong impression, for in the hands of a trained operator it is a powerful and sophisticated aid to computation. Some appreciation of the power of the abacus can be gained by noting the fact that in 1947 Kiyoshi Matsuzake of the Japanese Ministry of Communications used a soroban (the Japanese version of the abacus) to best Private Tom Wood of the United States Army of Occupation, who used the most modern electrically driven mechanical calculating machine, in a contest of speed and accuracy in calculation. The contest consisted of simple addition and subtraction problems, adding up long columns of many-digit numbers, and multiplication of integers. Matsuzake clearly won in four out of the five contests held, being only just beaten out by the electrically driven calculator when doing the multiplication problems. Although both men were highly skilled at their jobs, it should be pointed out that it took Matsuzake several years of special training in order to develop such a high order of skill at using the soroban and it is unlikely that the average abacus user would ever develop such speed and accuracy of operation. However, it does illustrate that, at least in the hands of even a moderately skilled operator, the abacus is far from being only an interesting toy.

The origin of the abacus is, literally, lost in the dusts of time. It likely started out as pebbles being moved over lines drawn in the dirt. Many cultures have used an abacus or counting board at some stage in their development, but as in most European countries, once paper and pencil methods were available the use of an abacus died out so completely that it is hard to find any cultural memory of the abacus being an important part of the arithmetic process. Today we tend to think of the abacus as a Far Eastern device, only because that is one of the few places where its use is still noticeable. In fact the abacus, in its present form, was only introduced into China in historical times (about A.D. 1200) and was taken from there to Korea (about A.D. 1400) and then to Japan (about A.D. 1600).

Although we know that the abacus was in general use in Europe until only about 250 years ago, we have remarkably little physical evidence of its presence, particularly from the earliest Greek and Roman times. What evidence we do have is usually in the form of quotations from the ancient writers. For example, Demosthenes (circa 384 B.C.– circa 322 B.C.) wrote of the need to use pebbles for calculations that were too difficult to do in your head. The use of the abacus was not confined to the Old World. We know very little about the various forms of abacus used by the Indians of North and South America, but we do know that some of these groups used the device. In 1590 a Jesuit, Joseph de Acosta, recorded some facts about the Inca culture that would indicate the common use of an abacus:

> In order to effect a very difficult computation for which an able calculator would require pen and ink . . . these Indians make use of their kernels of grain. They place one here, three somewhere else and eight I know not where. They move one kernel here and three there and the fact is that they are able to complete their computation without making the smallest mistake. As a matter of fact, they are better at calculating what each one is due to pay or give than we should be with pen and ink.[1]

It would seem likely that a number of North American Indian cultures were advanced enough to require some form of calculating device to be in use but almost no records remain of anything even as primitive as de Acosta's description. It is possible that the abacus was being used by some groups but that very few Europeans were concerned with recording anything except their own conquest of these Indian cultures.

The European Abacus

One of the few interesting bits of physical evidence for the early use of the table abacus comes from Greece. It is an actual abacus table, found on the island of Salamis (see Figure 1.1) just a few miles off the Greek coast near Piraeus. The Salamis abacus is now broken into two pieces, but was once a large marble slab about 5 feet long and 2 feet 6 inches wide. There is no indication of when it might have been made. From its size, it must have been used in some large public institution, perhaps as a bank or money changer's table. We know very little about how it may have been used except that it seems to be designed for counters to be placed on or between the various lines and the inscriptions appear to refer to numerical values and to certain types of coins, such as drachmae, talents, and obols. It has been speculated by many different people that the spaces between the five separate lines at one end of the abacus are intended for calculations involving fractions of the drachma.

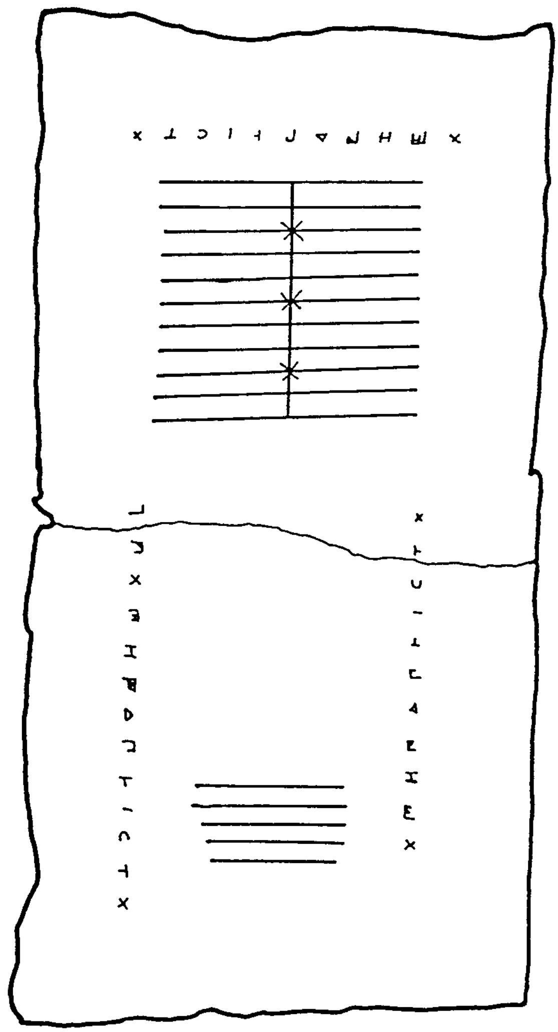


Figure 1.1. A drawing of the Salamas Abacus.

The word *abacus* itself can be of some help in determining the origins of the European version. The manipulation of pebbles in the dust, or the use of a finger or stylus in fine dust or sand spread upon a table, is known to have been used as an aid to calculation from very early times. The Semitic word *abaq* (dust) is thought by many to be the root of our modern word *abacus*. From the Semitic, the word seems to have been adopted by the Greeks who used *abax* to denote a flat surface or table upon which to draw their calculating lines. The term then appears to have spread to the Romans who called their table an *abacus*.

The term *abacus* has meant many different things during its history. It has been applied to the simple dust table, or wax tablet, which was generally used only as a substitute for pen and ink, as well as to the various forms of table abacus and different wire and bead arrangements used in the Far East. Because most early arithmetic was done on the abacus, the term became synonymous with 'arithmetic' and we find such oddities as Leonardo of Pisa (Fibonacci) publishing a book in 1202 called *Liber Abaci* (*The Book of the Abacus*), which did not deal with the abacus at all but was designed to show how the new Hindu-Arabic numerals could be used for calculation. In Northern Europe, the phrase *Rechnung auf der linien* (calculating on the lines) was in common use as a term meaning "to do arithmetic" even long after the use of the abacus had been abandoned.

Several of our modern mathematical and commercial terms can be traced to the early use of the table abacus. For example, the Romans used small limestone pebbles, called *calculi*, for their abacus counters; from this we take our modern words *calculate* and *calculus*. A more modern example comes from the fact that in England the table abacus was generally referred to as a *counting board* or simply as a *counter;* of course every merchant would have a counter in his shop upon which to place the goods being purchased and upon which the *account* could be calculated.

By the thirteenth century the European table abacus had been standardized into some variant of the form shown in the diagram below. It consisted of a simple table, sometimes covered by a cloth, upon which a number of lines were drawn in chalk or ink. The lines indicated the place value of the counters: the bottom line representing units and each line upwards representing ten times the value of the line below. Each space between the lines counted for five times that of the line below it. No more than four counters could be placed on a line and no more than one in any space. As soon as five counters appeared on a line, they were removed and one placed in the next higher space; if two appeared in a space, they were removed and one placed on the next higher line. When performing a computation on the table abacus, any counters in a space were considered to be grouped together with those on the line below: the use of the space simply being a device to keep the eye from being confused by having a large number of counters on one line. A cross or star was usually placed next to the fourth (thousands) line to guide the eye, much as we use a comma today to mark off groups of three digits. An example of such an abacus is shown in Figure 1.2.

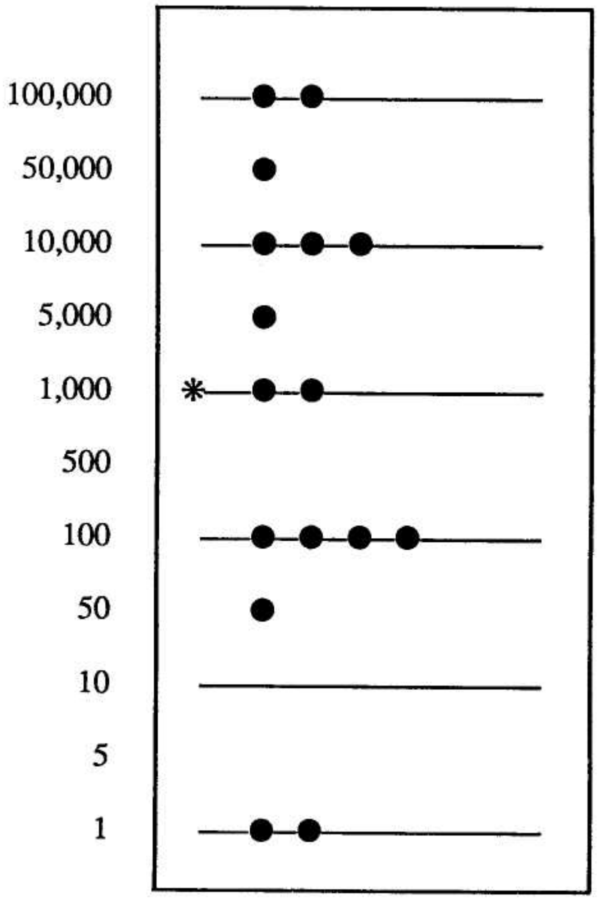


Figure 1.2. Table abacus set out to represent 287,452.

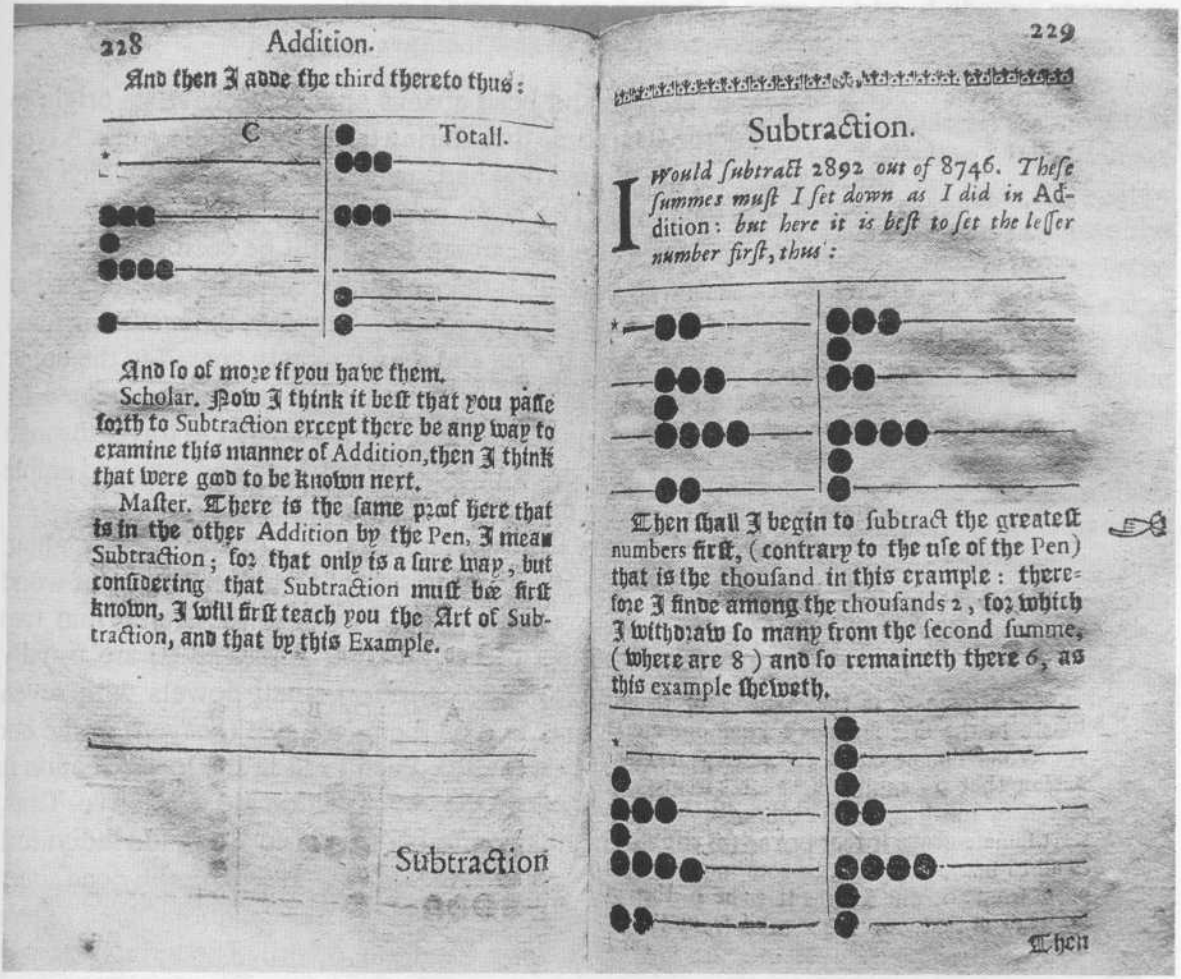
Very few of these *reckoning tables* still exist. We know that they once existed in great numbers for they are often mentioned in wills and in household inventories, but, being a common object, nobody thought to preserve them and only a handful are known still to exist in various museums.

By the thirteeth century the counters had changed from the simple pebbles used in earlier days into specially minted coinlike objects. They first appeared about 1200 in Italy, but because it was there that the use of Hindu-Arabic numerals first replaced the abacus, the majority of the counters now known come from north of the Alps. These coinlike counters were cast, thrown, or pushed on the abacus table, thus they were generally known by some name associated with this action. In France they were called *jetons* from the French verb *jeter* (to throw), while in the Netherlands they were known as *werpgeld* (thrown money). The older English usage of *to cast up an account* or *to cast a horoscope* also illustrates the mode of operation of a good abacist.

The counters, now commonly called *jetons*, are still to be found in quite large numbers. This is not surprising when you realize that the average numerate man would possess at least one set of copper jetons while a merchant would likely have several. Individuals possessing larger wealth or authority in the community would often have their jetons struck in silver with their coat of arms or portraits as the decoration.

The table abacus was used extensively in Britain even after it had been abandoned by the majority of people on the Continent. Illustrated in Figure 1.3 is one page from the first widely used printed book on arithmetic in the English language. This book, by Robert Recorde, was in print from 1542 right up to the start of the 1700s. It clearly shows (besides two errors in the illustration which are left as a puzzle for the reader) that abacus arithmetic was being taught to school children throughout this period.

The illustration for Recorde's book clearly shows the usual method of working a table abacus. For addition the two numbers were simply set down side by side and the two groups of jetons were simply moved together to accomplish the addition. Subtraction was slightly more difficult but was easily accomplished especially when one was able to literally "borrow" a jeton from a higher valued row in order to accomplish the process. The methods for multiplication and division were slightly different in various parts of Europe, but they



228 Addition.

And then I adde the third thereto thus:

C | Totall.

And so of more if you have them.

Scholar. Now I think it best that you passe forth to Subtraction except there be any way to examine this manner of Addition, then I think that were good to be knowne next.

Master. There is the same proofe here that is in the other Addition by the Pen, I mean Subtraction; for that only is a sure way, but considering that Subtraction must bee first knowne, I will first teach you the Art of Subtraction, and that by this Example.

Subtraction

229

Subtraction.

I would subtract 2892 out of 8746. These summes must I set down as I did in Addition: but here it is best to set the lesser number first, thus:

Then shall I begin to subtract the greatest numbers first, (contrary to the use of the Pen) that is the thousand in this example: therefore I finde among the thousands 2, for which I withdraw so many from the second summe, (where are 8) and so remaineth there 6, as this example sheweth.

Then

Figure 1.3. A page from Robert Recorde's book on arithmetic.

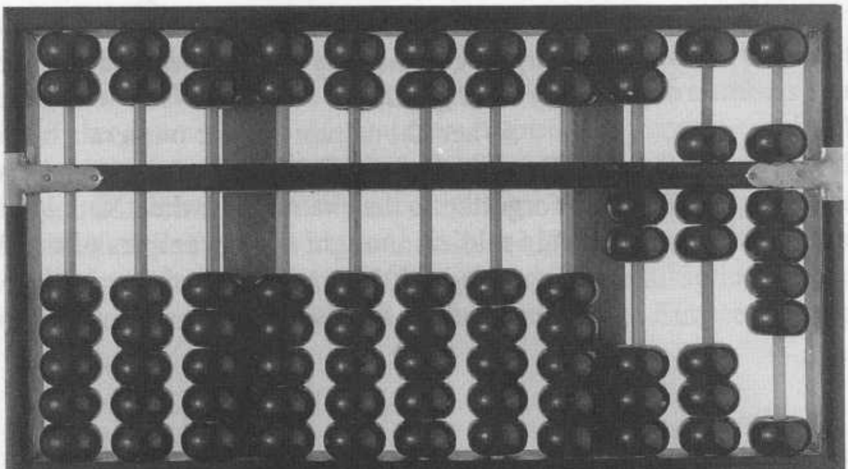
largely retained the doubling and halving processes that were started by the Egyptians.

When the Hindu-Arabic numerals became firmly established in Europe, the use of the table abacus died out completely. Its use was forgotten to the extent that, when Napoleon invaded Russia in 1812, his soldiers brought back examples of the Russian abacus as being a curiosity of the area; this was at a time when their own great-grandfathers had been daily users of the device in France.

The Abacus in the Orient

The oriental wire and bead abacus appears to have its origin in the Middle East some time during the early Middle Ages. A type of abacus was developed that had several wires, each of which was strung with ten beads. The Turks called this a *coulba*, the Armenians a *choreb*, and the Russians, where it can still be seen in use today, referred to it as a *stchoty*. This device almost certainly entered the Far East through the standard trade routes of the day, the merchant class being the first to adopt its use and then it slowly spread to the upper levels of society. Its introduction may well have been helped by international traders, such as Marco Polo, who had to travel through several different countries on their way to China and thus had ample opportunity to pick up different techniques along the way.

By the time it was firmly entrenched in Chinese society, about the year A.D. 1300, the abacus consisted of an oblong frame of wood with a bar running down its length, dividing the frame into two compartments. Through this bar, at right angles to it, are usually placed seventeen (but sometimes more) small dowels with seven beads strung on each one, two on the upper side (heaven) of the bar and five on the lower side (earth). Each bead in the lower section is worth one unit, while those in the upper section are worth five. Thus, it is possible to represent any number from 1 to 15 on the individual dowels, although anything greater than 9 would naturally occur only as an intermediate result in the process of a calculation. The Chinese called this device a *swan pan* (counting board). The term *swan* was derived from an older term meaning to "reckon with the rods"—a reference to an earlier oriental technique of using short bamboo rods to represent numbers on a flat calculating board (Figure 1.4).



From China the concept of a wire and bead abacus spread to Japan. Again it was likely the merchant class who actually spread the idea, for there was a great deal of trade going on between the two countries during the period A.D. 1400-1600. It is entirely possible that the soroban was being used in Japan for at least one hundred years before it was officially noticed by the ruling classes some time about 1600. At that time, the rulers of Japan were known to despise the lower classes; any knowledge of business affairs, or even of the value of the different coins, on the part of the nobility was considered a sign of inferior breeding. The soroban generally resembles the swan pan, except that there is only one bead in heaven and four in earth, and the beads themselves have been changed in shape to provide a sharper edge so that the operators fingers made better contact for flipping them up and down the dowels (Figure 1.5). These changes meant that the Japanese operator had to be a little more aware of how to work quickly with additions or subtractions, which may require a carry, or borrow, to or from the next column. It is, perhaps, with the soroban that the abacus reached its ultimate development. As was pointed out earlier, a well-trained soroban operator can compete with an electrically driven, four-function, mechanical calculator as far as speed and accuracy are concerned.

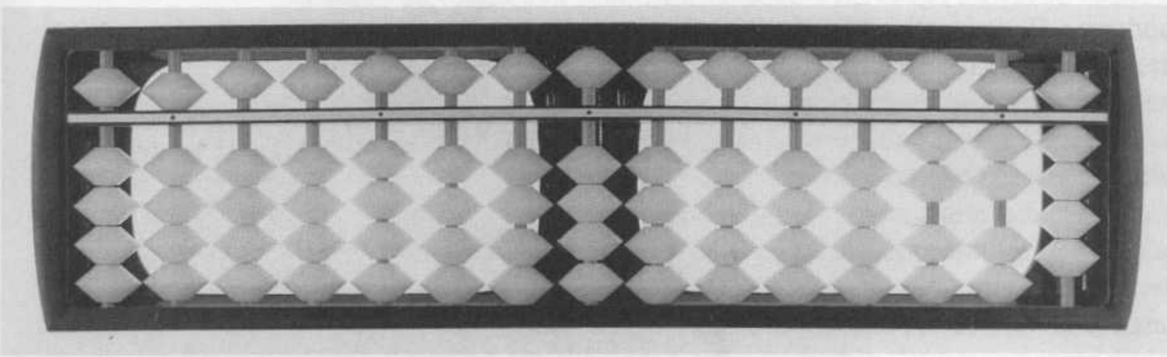


Figure 1.5. A Japanese soroban.

Figure 1.4. A Chinese swan pan.

Calculating Aids

Napier and His Bones

The Scottish Reformation was just starting as John Napier (Figure 1.6) was born in 1550 and the upheavals that it caused added to the misery of both the nobles and the common folk alike. In the middle of the sixteenth century, Scotland was torn apart by both political and religious strife, with war between the different groups being a constant occurrence. The cultural level of the time is said to have seldom risen above that of barbarous hospitality. Before Napier's time, Scotland had produced several men of note in the field of literature but only one in science, the thirteenth-century mathematician Michael Scott. With the study of academic subjects being held in low regard, it is very surprising that some of the most fundamental advances in mathematics and computation should have come out of this environment.

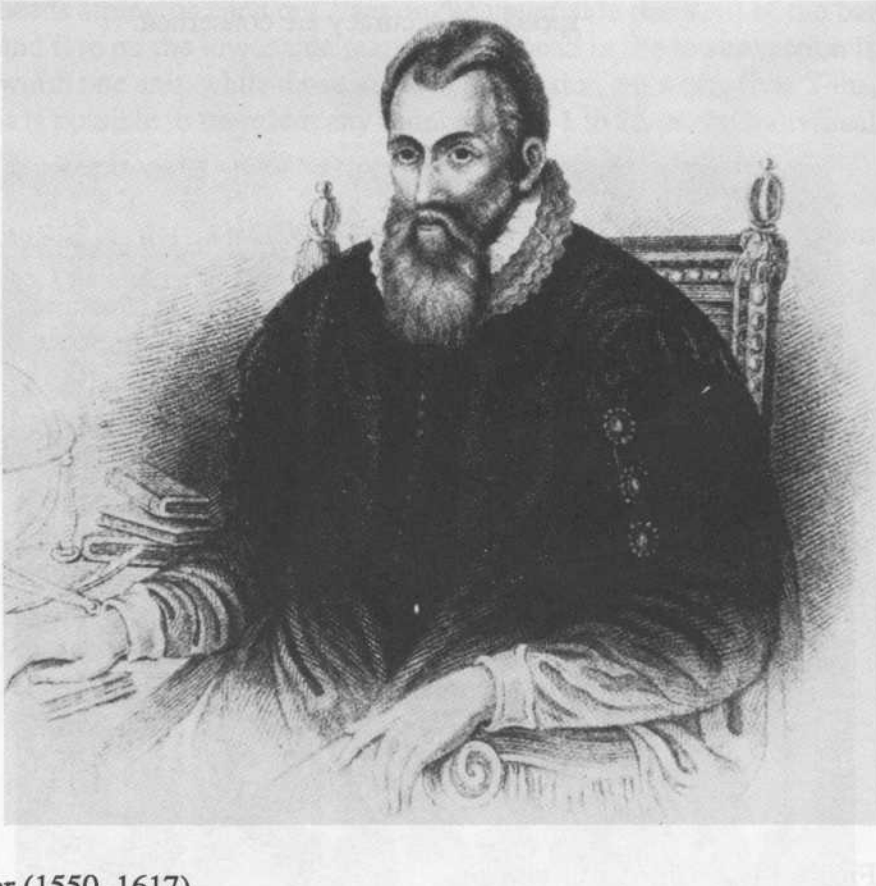


Figure 1.6. John Napier (1550–1617).

Napier was born near Edinburgh, but that is almost all we know of his early life. His father was one of the first people to take up the cause of the Protestant movement in Scotland and, presumably, he influenced John from his earliest days to believe that the pope was the sole bar to the salvation of all humanity. Certainly John held this belief right up to the time he died in 1617.

Napier is best known for his invention of logarithms, but he spent a large part of his life devising various other schemes for easing the labor involved in doing arithmetic. One of the best known of these devices is his *Rabdologia*, or as they are more commonly known *Napier's Bones*. The name *bones* arose from the fact that the better quality sets were constructed from horn, bone, or ivory. Various authors have preferred to call them "numbering rods," "multiplying rulers," or even "speaking rods," but the name *bones* just refused to to die out. Today they are usually considered a mere curiosity.

Napier did not at first consider this invention worthy of publication; however, several friends pressed him to write it up, if only to avoid others claiming it as their own. His descriptions appeared in 1617, the year of his death and three years after the publication of his description of logarithms, in a small book entitled *Rabdologia*.

The idea for the bones undoubtedly came from the Gelosia method of doing multiplication. This method is known to be very old; it likely developed in India and there are records of its use in Arabic, Persian, and Chinese societies from the late Middle Ages. The method was introduced into Italy sometime in the fourteenth century, where it obtained its name from its similarity to a common form of Italian window grating. The method consists of writing down a matrixlike grid, placing one digit of the multiplicand at the head of each column and one digit of the multiplier beside each row, the product of each row and column digit is then entered in the appropriate box of the matrix—the tens digit above the diagonal and the units digit below. The final product is obtained by starting in the lower right-hand corner and adding up the digits in each diagonal with any carry digits being considered as part of the next diagonal. Figure 1.7 illustrates the Gelosia method, showing 456 multiplied by 128 with the product (058368) being read off starting from the upper left-hand corner.

Napier's bones are simply a collection of strips of all possible columns of this Gelosia table as is shown in Figure 1.8. To perform the multiplication of 456 by 128 one would select the strips headed 4, 5, and 6, place them side by side, read off the partial products of

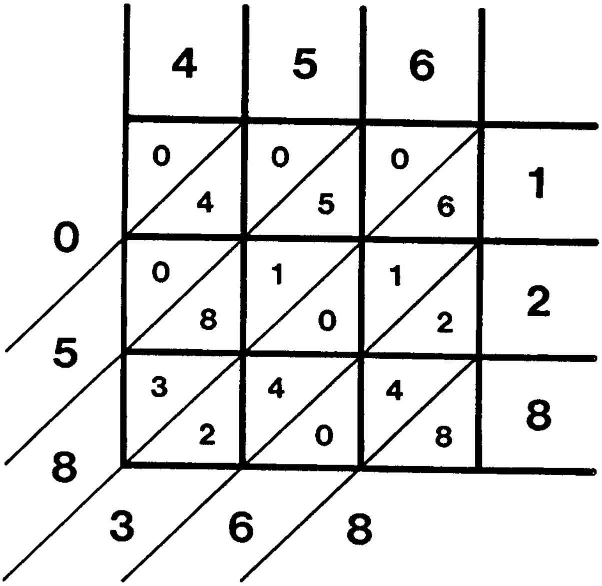


Figure 1.7. The Gelosia method of multiplication.

456 times 1, 456 times 2 and 456 times 8 (by adding up the digits in each parallelogram to obtain each digit of the partial product), and then add together the partial products. Division was aided by the bones in that multiples of the divisor could be easily determined, saving time that would normally be spent in trial multiplication.

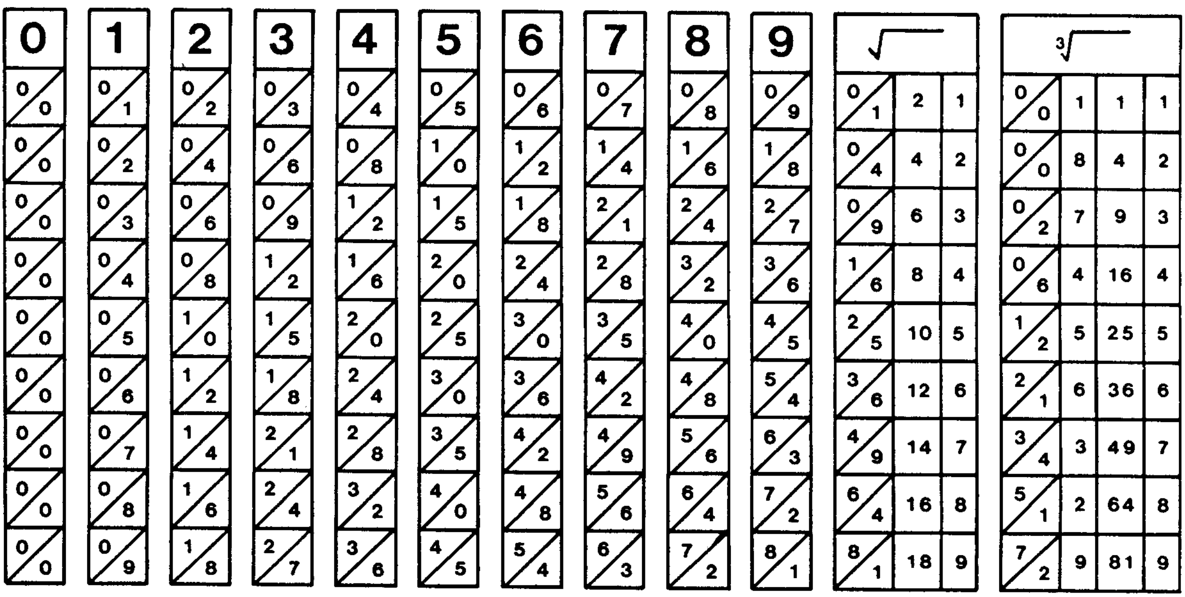


Figure 1.8. A modern set of Napier's bones.

The use of Napier's bones spread rapidly, and, within a few years, examples could be found in use from Europe to China. It is likely that the two Jesuits, Gaspard Schott and Athanasius Kircher, were partially responsible for their spread, particularly to China, where two other Jesuits held office in the Peking Astronomical Board. Both Schott and Kircher were German mathematicians during the time when the Jesuit order was sending its technically trained members around the world as missionaries for both the Christian faith and the wonders of European technology.

Schott was aware of the physical problems involved in using a standard set of arithmetic bones: such things as locating the correct bones, having some convenient device to ensure they line up correctly, etc. Several others had suggested incorporating Napier's bones into some form of mechanical assembly but none of them had published any of their ideas, so Schott was left on his own to invent a similar device. The result was a series of cylinders with a complete set of Napier's bones inscribed on each, the individual bones running the length of the cylinder. Several of these cylinders were then mounted in a box so they could be turned and any individual bone could be examined through slits cut in the top of the box. Figure 1.9 shows a photograph of Schott's device, the top of the box containing an addition table to aid the operator.

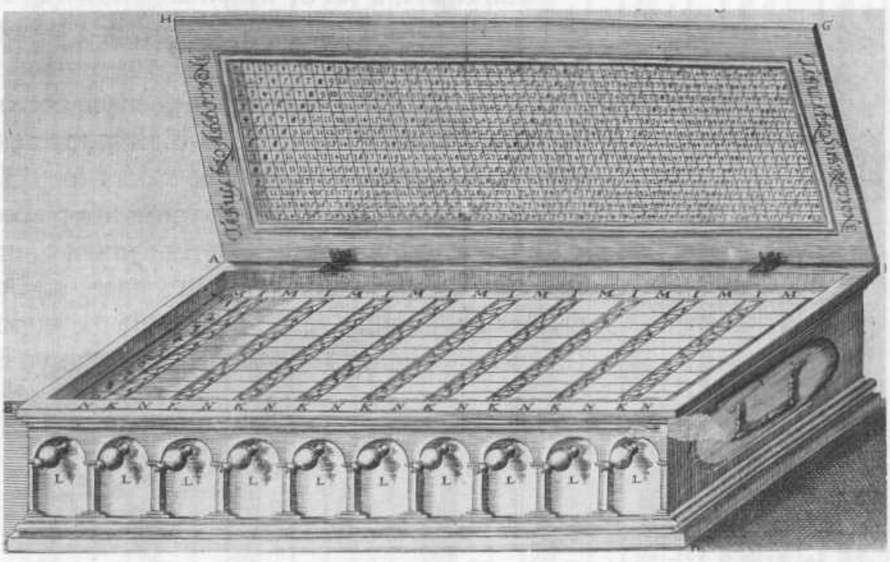


Figure 1.9. Gaspard Schott's version of Napier's bones.

Although it was an interesting attempt at making the bones easier to use, the system proved to be a failure. The parallelograms containing the digits to be added together span two adjacent bones and the space required to mount the cylinders meant that these digits were widely separated. This led to a greater tendency to make mistakes and the device was soon abandoned. Schemes, similar to Schott's, were tried by different people in different countries (most notably by Pierre Petit, the French mathematician and friend of Pascal) but they all failed for the same reason.

The final chapter in the development of Napier's bones as a computational instrument took place in 1885 when, at the French Association for the Advancement of Science meetings, Edouard Lucas presented a problem on arithmetic that caught the attention of Henri Genaille, a French civil engineer working for the railway. Genaille, who was already quite well-known for his invention of several different arithmetic aids, solved Lucas's problem and, in the process, devised a different form of Napier's bones. These "rulers" eliminated the need to carry digits from one column to the next when reading off partial products (Figure 1.10). He demonstrated these rulers to the association in 1891. Lucas gave these rulers enough publicity that they became quite popular for a number of years. Unfortunately he never lived to see their popularity grow, for he died, aged 49, shortly after Genaille's demonstration.

The rulers, a set of which are shown in Figure 1.10, are similar in their use to a standard set of Napier's bones. There is one ruler for each digit from 0 to 9. Each ruler is divided into nine sections with several digits inscribed in each section, and one or two arrows point to the left towards a particular digit in the next ruler. In order to find the product of 3271 by 4, the rulers for 03271 (note the need for always having a leading zero ruler) are placed side by side. Starting with the fourth section of the right-most ruler, you select the digit at the top of this section (4 in this case) and then simply follow the arrows towards the left, reading off the digits as you come to them (the product being 13084 in the case shown in Figure 1.11).

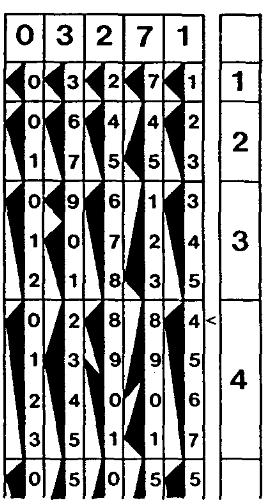
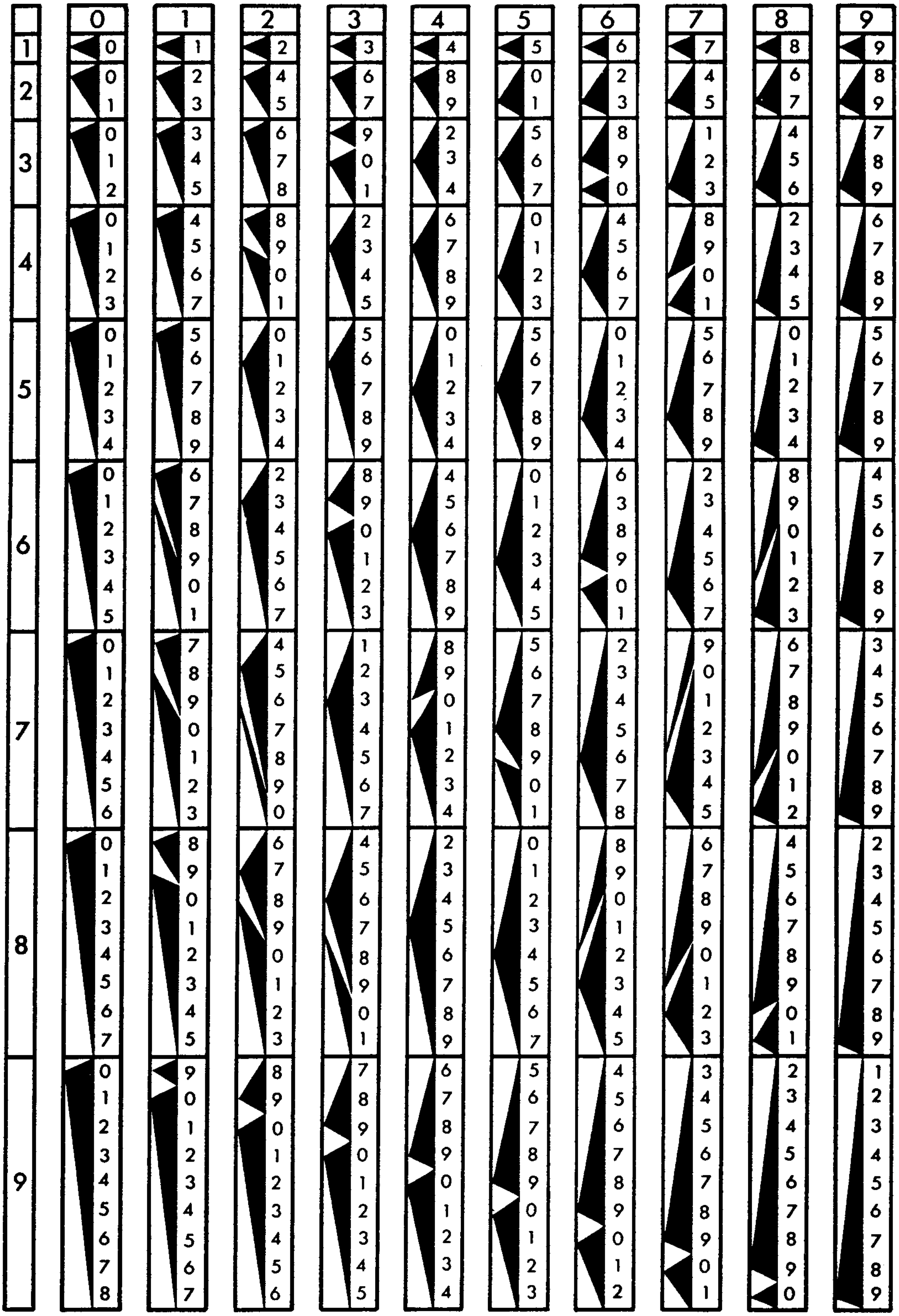


Figure 1.11. The rulers being used to find 3271 times 4.

Figure 1.10. A set of the Genaille-Lucas rulers.



Once the problem of eliminating the carry digits had been solved by Genaille, the creation of a specific set of rulers for division was quickly accomplished. The division rulers are similar to the multiplication ones except that the large arrows are replaced by a multitude of smaller ones. Figures 1.12 and 1.13 show a complete set of division rulers together with an example of how they could be used to divide the number 6957 by 6.

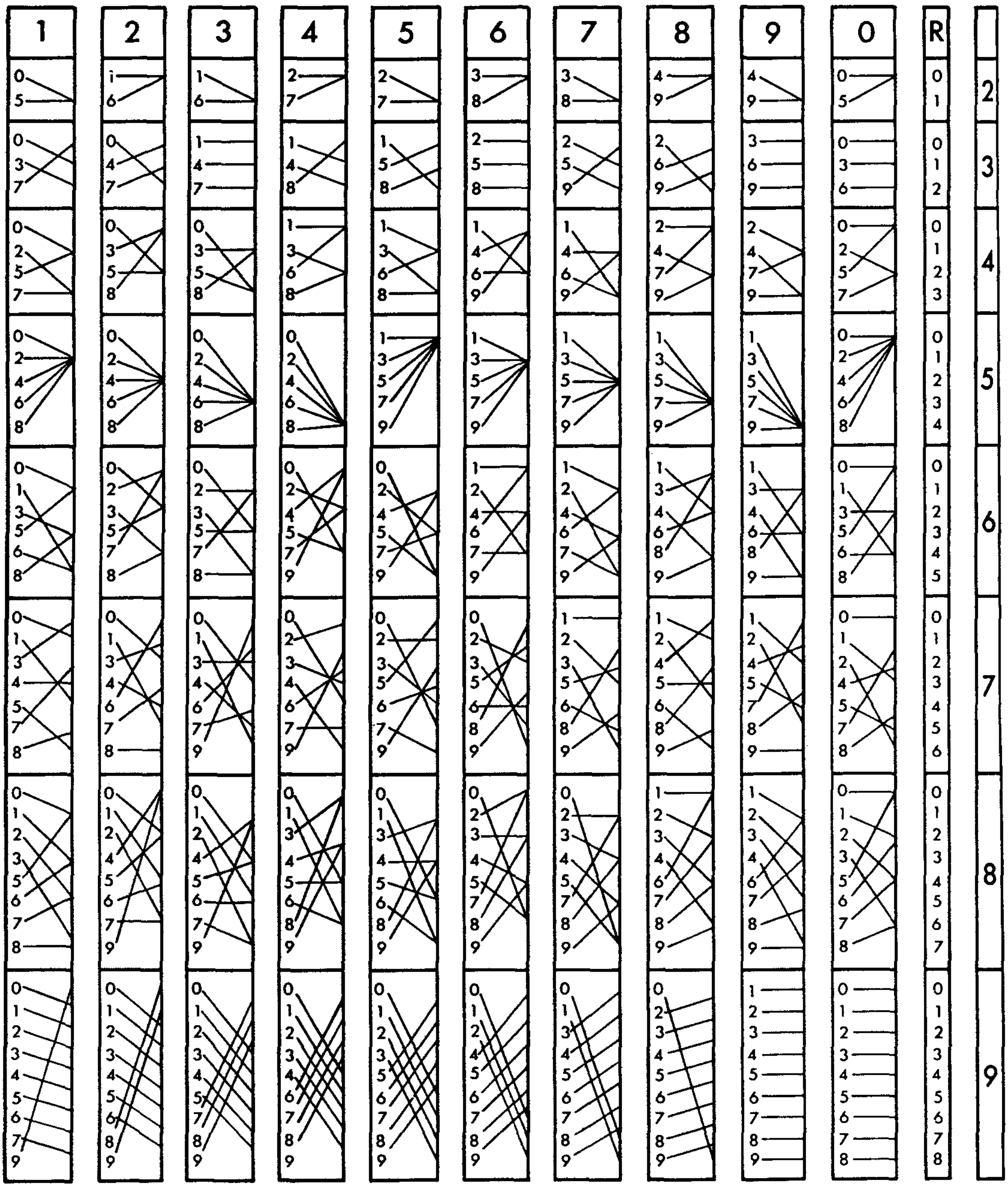
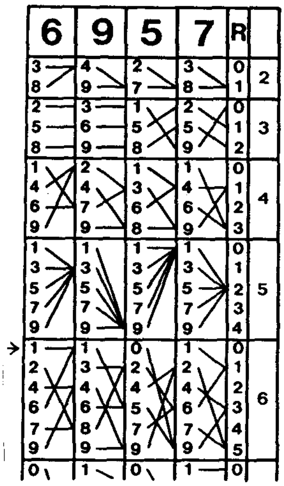


Figure 1.12. A set of the Genaille-Lucas division rulers.



6957 ÷ 6 = 1159 R3

Figure 1.13. Genaille-Lucas division rulers used to divide 6957 by 6.

Note that a special ruler (marked *R*) must be placed on the right-hand side of the set in order to determine the remainder, if any, of the division operation. The division rulers are used in the opposite direction from the multiplication ones. In order to divide the number by 6, you start at the left hand side of the sixth section with the topmost digit (1 in the case shown here) and proceed to the right, following the arrows and reading off the digits as they are encountered (1159 with a remainder of 3).

In the era before the mechanical desk-top calculating machine industry had been developed, these simple instruments were one of the two main forms of computational assistance for anyone engaged in scientific or business calculations more complex than elementary addition and subtraction. The other main computational aid, like these various forms of Napier's bones, also began with some pioneering work of John Napier and is discussed below.

Logarithms

Many writers have suggested that the invention of logarithms came like a bolt from the blue, with nothing leading up to them. This is not exactly the case because, like almost every other invention, examples can be found of parallel development by other people. John Napier is always given the credit for logarithms because these other developments were either left unpublished or, in some cases, not recognized for what they were at the time.

The major computational problems of Napier's time tended to involve astronomy, navigation, and the casting of horoscopes, all of which are interrelated. These problems led to a number of sixteenth-century scientists devoting their time to the development of trigonometry. About twenty-five years before Napier published his description of logarithms, the problem of easing the workload when multiplying two sines together was solved by the *method of prosthaphaeresis*, which corresponds to the formula:

$$\sin a \times \sin b = [\cos(a - b) - \cos(a + b)]/2$$

Once it had been shown that a rather nasty multiplication could be replaced by a few simple additions, subtractions, and an elementary division by 2, it is entirely likely that this formula spurred scientifically oriented individuals, including Napier, to search for other methods to simplify the harder arithmetical operations. In fact several other such formulae were developed during Napier's time, but only the method of prosthaphaeresis was of any real use, except in special circumstances. We know that Napier knew of, and used, the method of prosthaphaeresis, and it may well have influenced his thinking because the first logarithms were not of numbers but were logarithms of sines.

Another factor in the development of logarithms at this time was that the properties of arithmetic and geometric series had been studied extensively in the previous century. We now know that any numbers in an arithmetic series are the logarithms of other numbers in a geometric series, in some suitable base. For example, the following series of numbers is geometric, with each number being two times the previous one:

natural numbers 1 2 4 8 16 32 64 128 256 512 1024.

And the series below is an arithmetic one whose values are the corresponding base 2 logarithms:

logarithms 0 1 2 3 4 5 6 7 8 9 10.

It had long been known that if you take any two numbers in the arithmetic progression, say 3 and 4, their sum, 7, would indicate the position of the term in the geometric series that is the product of the terms in the corresponding positions of the geometric series, e.g., $3 + 4 = 7$ and $8 \times 16 = 128$ (the third times the fourth = the seventh). This is starting to look very much like our own conception of logarithms as being the powers to which some base number is raised, a concept that was not understood in Napier's time. Often the use of a good form of notation will suggest some basic mathematical principle. Our use of indices to indicate the power to which a number is being raised seems to have an obvious connection with logarithms, but without this form of notation, the connection is vague at best.

John Napier came at the idea of logarithms not by algebra and indices but by way of geometry. When first thinking about this subject, he used the term *artificial number* but later created the term *logarithm* from a Greek phrase meaning "ratio number." He decided on this term because his logarithms were based on the concept of points moving down lines in which the velocity of one point was based on the ratio of the lengths of the line on either side of it.

We know almost nothing about how long Napier worked before he felt that the idea of logarithms was sufficiently refined to be worthy of publication, but in July of 1614 he published a small volume of fifty-six pages of text and ninety pages of tables entitled *Mirifici Logarithmorum Canonis Descriptio*. At best, it is translated as *Description of the Admirable Cannon (Table) of Logarithms*. It was common in those days to dedicate a book to a nobleman, often in the hope that some patronage would result. Unfortunately Napier had the bad luck to dedicate the *Descriptio* to the then Prince of Wales, who, when he later became King Charles I, was beheaded by Cromwell.

The *Descriptio* was just that, a description of the cannon or table of logarithms of sines, with the rules to be followed when using them to perform multiplication, division, or the computation of roots and powers. It contained a statement that, if these tables were accorded the reception that Napier hoped, he would describe in some future publication exactly how they were discovered and the methods used to calculate them.

Our story now shifts to London, where one of the most famous English mathematicians of the day, Henry Briggs (1561-1631), was Professor of Geometry at Gresham College. By the early years of the 1600s his reputation had spread far enough that people like Johann Kepler were consulting him on the properties of the ellipse. In the later months of 1614 he obtained a copy of Napier's *Descriptio* and, by March of the following year wrote that

> Napier, lord of Markinston, hath set my head and hands at work with his new and admirable logarithms. I hope to see him this summer, if it please God; for I never saw a book which pleased me better, and made me more wonder.[2]

Briggs immediately began to popularize the concept of logarithms in his lectures and even began to work on a modified version of the tables. Several years later, in 1628, Briggs's newly calculated logarithms were published and he stated in the Latin preface

> That these logarithms differ from those which that illustrious man, the Baron of Merchiston published in his Cannon Mirificus must not

> surprise you. For I myself, when expounding their doctrine publicly in London to my auditors in Gresham College, remarked that it would be much more convenient that 0 should be kept for the logarithm of the whole sine.... And concerning that matter I wrote immediately to the author himself; and as soon as the season of the year and the vacation of my public duties of instruction permitted I journeyed to Edinburgh, where, being most hospitably received by him, I lingered for a whole month.[3]

What Briggs was suggesting was that the base of the logarithms should be changed in order to make them easier to use. Napier had evidently already seen the same thing, for as Briggs states:

> But as we held discourse concerning this change in the system of Logarithms, he said, that for a long time he had been sensible of the same thing, and had been anxious to accomplish it, but that he had published those he had already prepared, until he could construct tables more convenient, if other weighty matters and his frail health would suffer him so to do. But he conceived that the change ought to be effected in this manner, that 0 should become the logarithm of unity, and 10,000,000,000 that of the whole sine; which I could but admit was by far the most convenient of all. So, rejecting those which I had already prepared, I commenced, under his encouraging counsel, to ponder seriously about the calculation of these tables; and in the following summer I again took journey to Edinburgh, where I submitted to him the principal part of those tables which are here published, and I was about to do the same even the third summer, had it pleased God to spare him so long.[4]

The result of these changes was to create the common (base 10) logarithms that we know today.

Henry Briggs never did finish his complete recalculation of Napier's logarithms. His tables, first published in 1624, contained the logs of the numbers from 1 to 20,000 and from 90,000 to 100,000 all calculated to 14 decimal places. There are 1161 errors in these original tables, or just under 0.04 percent of the entries. Almost all of them are simple errors of plus or minus 1 in the last decimal place; however, several more are printing or copying errors such as the printing of 3730 instead of 4730, but these are easily seen by users of the tables because they stand out as being quite different from the surrounding entries.

The concept of logarithms spread rapidly. In the same year as Briggs's tables appeared, Kepler published his first set of logarithms and, a year later, Edmund Wingate published a set in Paris called *Arithmetique Logarithmique*, which not only contained logarithms for the numbers from 1 to 1000, but also contained Edmund Gunter's newly calculated log sines and log tangents. The first complete set of logarithms for the numbers from 1 to 101,000 was published by a Dutch printer, Adrian Vlacq (circa 1600-1667), who was noted for his ability at printing scientific works. He filled in the sections missing from Briggs's work, and published the whole table in 1628. Vlacq's tables were copied by many others in later years. Although the publishers seldom acknowledged the source of the logarithms, it was obvious where they came from because Vlacq's original errors were copied along with the correct logarithms. It was not until the first quarter of the nineteenth century, when Charles Babbage published his famous log tables, that correct sets of tables were readily available.

Within twenty years of the time that Briggs's tables first appeared, the use of logarithms had spread worldwide. From being a limited tool of great scientists like Kepler, they had become commonplace in the schoolrooms of the civilized nations. Logarithms were used extensively in all trades and professions that required calculations to be done. It is hard to imagine an invention that has helped the process of computation more dramatically than has logarithms, the one exception being the modern digital computer. During a conference held in 1914 to celebrate the three hundredth anniversary of the publication of the *Descriptio*, it was estimated that, of all the calculation done in the previous three hundred years, the vast majority had been done with the aid of logarithms.

The Slide Rule

Although logarithms were usable in the form in which Napier invented them, it was the work of Henry Briggs that actually made them easier to use. Briggs's work naturally came to the notice of Edmund Gunter, another professor at Gresham College, who was a very practically minded teacher of astronomy and mathematics. Gunter was primarily interested in the problems of astronomy, navigation, and the construction of sun dials (the only reasonable method of telling time in his day), all of which required large amounts of calculation involving trigonometric elements. Because of the trigonometric content of these problems, the logarithm tables being produced by Briggs were only of marginal help, so Gunter sat down and completed the calculations for tables of the logarithms of sines and the logarithms of tangents for each minute of the quadrant. These eight figure tables were published in 1620 and did much to relieve the burden of calculation for finding one's position at sea.

Gunter had some earlier experience in the development of calculating instruments, having been one of the major figures in the perfection of an instrument known as a *sector*. This device used a pair of dividers to measure off different values inscribed along several different linear scales. This experience soon led him to realize that the process of adding together a pair of logarithms could be partially automated by engraving a scale of logarithms on a piece of wood and then using a pair of dividers to add together two values in much the same way as he would have done when using a sector. Not only did this method eliminate the mental work of addition, but it also removed the necessity for the time-consuming process of looking up the logarithms in a table. Gunter's piece of wood soon became known as Gunter's Line of Numbers. Its use spread rapidly through England and was quickly popularized on the Continent.

Gunter's Line of Numbers consisted of a simple piece of wood, about two feet long, (often the shaft of a cross-staff, a simple navigational sighting instrument of the time) marked off with a logarithmic scale, much the same way as one axis of a piece of logarithmic graph paper is marked today. If he wished to multiply A times B, he would open up a pair of dividers to the distance from 1 to A on his line of numbers, putting one point of the compass on the point B, he would read off the number at which the other point sat. The accuracy was limited to two or three digits, depending on the care with which the instrument was used, but he had produced the first logarithmic analog device able to multiply two numbers together. Gunter would likely have added further refinements to his Line of Numbers, for he was a master at the design and use of instruments, but he died, aged 45, in 1626, before he was able to get enough time from his other duties to return to the subject of logarithmic calculating instruments. The next developments were left to a highly individualistic clergyman named William Oughtred.

William Oughtred (circa 1574-1660) was one of the leading mathematicians of his day. In 1604, after having taken a degree at Cambridge, he was appointed as the rector of a small parish in Surrey and, a few years later, was moved to the parish of Albury where he lived for the rest of his life. He was the bane of his bishop, being the subject of several complaints that he was a pitiful preacher because he never studied anything other than mathematics (which tends to make for dull sermons). In the days before regular scientific journals, information was published by sending it to several people who were known to be in regular contact with other scientific men—Athanasius Kircher, mentioned in connection with Napier's bones, and Fr. Martin Mersenne of Paris being the noted "postboxes" on the Continent, while William Oughtred was one of the main distribution points for England.

Oughtred was what we would now classify as a "pure" mathematician. Although he had a contempt for the computational side of mathematics and considered the people who used calculational instruments simply as "the doers of tricks," he was quite familiar with the mathematical instruments then available. There are records of his visiting Henry Briggs in 1610 and, while there, meeting Edmund Gunter, and discussing mathematical instruments with him at great length.

Oughtred noted that Gunter's Line of Numbers required a pair of dividers in order to measure off the lengths of logarithmic values along the scale and quickly came up with the idea that, if he had two such scales marked along the edges of the pieces of wood, he could slide them relative to each other and thus do away with the need for a pair of dividers. He also saw that if there were two disks, one slightly smaller than the other, with a Line of Numbers engraved around the edge of each, that they could be pinioned together at their centers and rotated relative to one another to give the same effect as having Gunter's scale engraved on two bits of wood.

Because of his general disdain for mathematical instruments he did not consider it worth his trouble, time, or effort to publish a description of how he had improved Gunter's Line of Numbers into a practical slide rule. He did, however, describe the system to one of his pupils, Richard Delamain, who was a teacher of mathematics living and working in London. Delamain used Oughtred's ideas quite openly and based his teaching on various methods of instrumental calculation.

In 1630 another of Oughtred's pupils, William Forster, happened to mention that in order to gain more accuracy when using Gunter's Line of Numbers he had resorted to using a scale six feet long and a beam compass to measure off the lengths. Oughtred then showed him how he could dispense with the beam compass by simply having two of Gunter's scales sliding over one another and also showed him a circular disk with Gunter's Line of Numbers marked off along the edge with two indices, like a pair of dividers, extending from the center. The latter device, which Oughtred called his "Circles of Proportion" (shown in Figure 1.14), he claimed to have invented sometime in 1622. Forster was so impressed that he demanded Oughtred publish a description of these inventions. Oughtred, still under the impression that these "playthings" were not suitable objects for the true mathematician, initially decided against it but, when Delamain's book appeared claiming them as his own invention, Oughtred agreed to publish and even let Forster translate his Latin into English so that the subject matter would be more widely distributed than if it had remained in academic Latin.

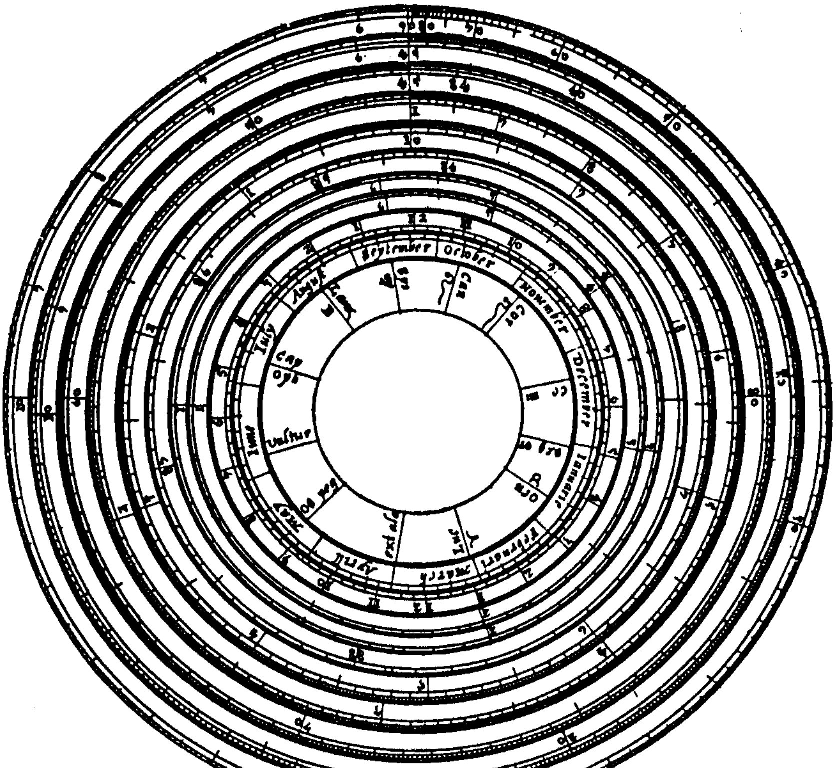


Figure 1.14. Oughtred's circles of proportion.

The slide rule may have been developed and publicized in the 1630s and obtained its current form as a movable slide between two other fixed blocks of wood about the middle 1650s, but very little use was actually made of the device for almost two hundred years. However several special slide rules were developed and became quite popular; for example, a special slide rule was created for the use of timber merchants, but the average educated man still clung to the older sector as his main calculating instrument.

James Watt, better known for his work on the steam engine, was responsible, at least in part, for one of the first really well-made slide rules in the very late 1700s. He had spent the early part of his life as an instrument maker at Glasgow University and so was familiar with the techniques of engraving accurate scales upon instruments. After he had set up a workshop for his steam engine business in Soho, Birmingham, he discovered that he needed a device to let him perform quick calculations concerning the volumes and power levels of various engines. He devised a simple slide rule consisting of one sliding piece between two fixed stocks (a design that had been in use for a considerable period of time), carefully engraved the face with four basic scales, and put tables of various constants on the back. His rule was accurate enough that others soon requested copies for themselves and Watt manufactured this so-called Soho Slide Rule for several years. Even with the example of the Soho Slide Rule, the general public seemed to ignore the power of the instrument. The great English mathematician Augustus De Morgan, when writing an article about the slide rule for the popular press in 1850, had to explain that

> for a few shillings most persons might put into their pockets some hundred times as much power of calculation as they have in their heads.[5]

The big breakthrough for the slide rule came in 1850, when a nineteen-year-old French artillery officer, Amedee Mannheim (1831-1906), designed a very simple slide rule much like that manufactured by Watt, but added the movable double-sided cursor, which we think of as such an integral part of the slide rule today. This was not the first time that a movable cursor had been combined with the simple sliding logarithmic scales, indeed the first time had been almost two hundred years earlier on a slide rule designed for British naval use, but it had been largely ignored until Mannheim reinvented it. The cursor enabled fairly complex operations to be easily carried out on a simple, yet well-made, slide rule (Figure 1.15). Mannheim's design was adopted as the standard for the French artillery and, after a few years, examples of it began to appear in other countries. Mannheim survived his army service and was eventually appointed Professor of Mathematics at the celebrated École Polytechnique in Paris, a post that did nothing to harm the evergrowing reputation of his slide rule.

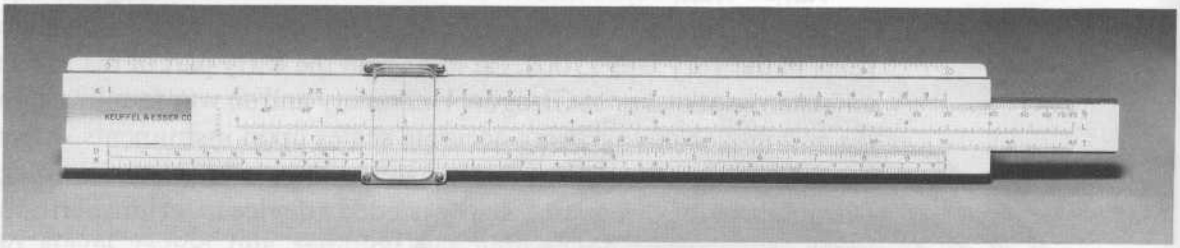


Figure 1.15. A modern version of the Mannheim slide rule. Courtesy Smithsonian Institution.

Despite the fact that the Europeans began to adopt the "slip stick" for many forms of quick calculation, it remained unpopular in North America until 1888, when several examples of the Mannheim design were imported. The North American market grew until, in 1895, there was enough of a demand that the Mannheim rules were manufactured in the United States. Even with a local source of manufacture, the slide rule was still not totally accepted in North America until the twentieth century. A survey in the journal *Engineering News* reported that, as late as 1901, only one-half of the engineering schools in the United States gave any attention at all to the use of the slide rule.

Once established, the progress of the slide rule was extremely rapid. Many different forms were produced by several different major manufacturers. The number of scales to be found on each instrument increased to the point that eighteen or twenty different scales were regularly engraved on the better quality instruments. Both sides of the rule were used and the center, sliding portion could often be turned over or completely replaced to provide even more combinations of scales. Special slide rules incorporating such things as a scale of atomic and molecular weights were created for chemists, and almost any branch of science or engineering could boast that at least one manufacturer produced a slide rule designed for their particular use. The accuracy of the slide rule was improved by several people who modified the basic form so that the logarithmic scales were wrapped around cylinders or into spirals. One device, known as Fuller's Slide Rule (Figure 1.16), was equivalent to a standard slide rule over eighty-four feet long, yet could be easily held in the hand. It was possible to work correctly to four figures, and sometimes even five, with this particular unit.

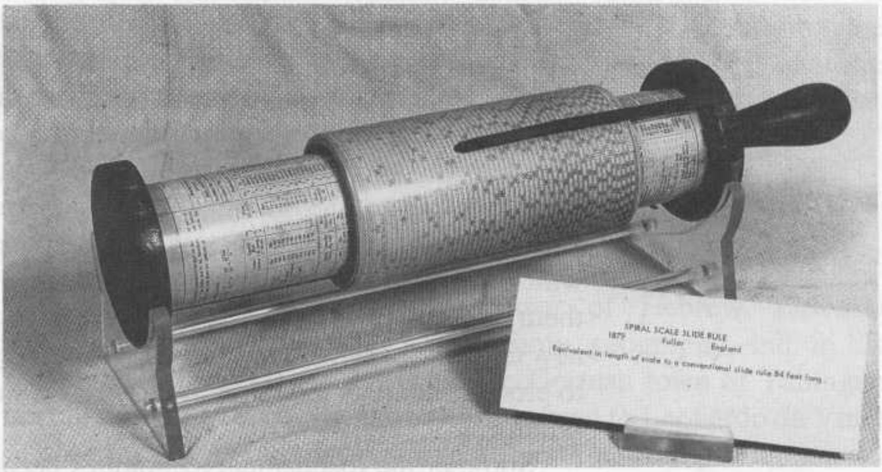


Figure 1.16. Fuller's slide rule.

The slide rule became a symbol that was often used to represent the advancing technology of the twentieth century. It was a status symbol for engineering students in the 1950s and 1960s and could almost always be found clipped to their belt as a statement of their chosen profession. It was, however, to be a transient symbol. The development of the hand-held electronic calculator offered many times the accuracy and convenience and the slide rule quickly sank into obscurity. The demise was so rapid that it is possible to find many examples of people who differ in age by only four or five years, one of whom relied entirely on the slide rule for all calculations required during university education, and the other, who took the same course of studies, would not know how to use it to multiply two numbers together. In a matter of a few years the major manufacturers of slide rules had to either transfer their expertise to other products or face bankruptcy.

Mechanical Calculating Machines

Introduction

Though the various analog instruments were capable of performing a great deal of useful arithmetic, the story of devices that ultimately led to fully automatic computation really starts with the invention and development of mechanical devices to perform the four standard arithmetic functions. By devising a system in which mechanical levers, gears, and wheels could replace the facilities of human intellect, the early pioneers in these devices showed the way towards the complete automation of the process of calculation. Needless to say the early efforts were very crude not because the inventors lacked the intelligence to construct better devices but because the technical abilities of the workmen and the materials with which they had to work were often not up to the demands put upon them by these new machines. There was also the problem that whole new techniques had to be invented in order to get mechanical devices to produce some of the motions required of them when doing simple arithmetic.

Some of the mechanical techniques became available about the start of the seventeenth century, when, in response to a demand for mechanical automata to amuse the rich, methods of producing various motions in mechanical systems were developed. The construction techniques were further advanced by the developing trade of the clock maker—several early computing machines were built by people trained in horological arts.

Most of the very early attempts at constructing a simple adding machine relied on the human operator to adjust the mechanism whenever a carry occurred from one digit to the next, much the same way as was done when using a table abacus. There is no point in detailing the development of this type of mechanism as they were all of the most elementary kind and, in general, only constructed from crude materials. The real development of mechanical computing machinery only began when people attempted to incorporate mechanisms to automatically deal with the problem of adding a carry from one digit to the next.

It used to be thought that Blaise Pascal invented the first adding machine to contain a carry mechanism; however, investigative work in the 1950s and 1960s showed that that honor belongs to Wilhelm

Schickard, who produced a machine about the year 1620, some twenty years before Pascal's attempt. It is quite possible that further investigation will reveal yet an earlier device, but nothing now suggests that any work of importance was done before Schickard. There are many stories of people creating adding machines before Schickard, some even as early as the year 1000. For example, the monk Gerbert (later Pope Sylvester II) is reputed to have developed some form of early calculating device, but it is almost certain that these legends refer to things like Gerbert's abacus rather than an actual mechanical device. Even if people like Gerbert did produce some form of mechanical mechanism, it is most unlikely that the technology have allowed anything to be produced matching the sophistication of the Schickard or Pascal machines.

The Machines of Wilhelm Schickard (1592-1635)

Wilhelm Schickard was Professor of Hebrew, Oriental languages, mathematics, astronomy, geography, and, in his spare time, a protestant minister in the German town of Tübingen during the early 1600s. He has been compared to Leonardo da Vinci in that they both had far-ranging interests and enquiring minds. Besides being an excellent mathematician, with some of his mathematical methods being in use until the later part of the nineteenth century, he was a good painter, a good enough mechanic to construct his own astronomical instruments, and a skilled enough engraver to provide some of the copper plates used to illustrate Kepler's great work *Harmonices Mundi*.

Figure 1.17. Wilhelm Schickard (1592-1635).

It is known that Schickard and Kepler not only knew each other but that they also worked together several times during their lives. It was one of these joint efforts that resulted in Schickard producing the first workable mechanical adding machine. Kepler and Schickard are known to have discussed John Napier's invention of logarithms and Napier's bones as early as 1617. During one of Kepler's visits to Tübingen he showed Schickard some of his new results and examples of Napier's bones and logarithms, which he had used in their calculation. This seems to have inspired Schickard to consider the design of a machine that would incorporate both a set of Napier's bones and a mechanism to add up the partial products they produced in order to completely automate the process of finding the product of two numbers.

On September 20, 1623, Schickard wrote to Kepler saying that

> what you have done in a logistical way (i.e., by calculation), I have just tried to do by mechanics. I have constructed a machine consisting of eleven complete and six incomplete (actually "mutilated") sprocket wheels which can calculate. You would burst out laughing if you were present to see how it carries by itself from one column of tens to the next or borrows from them during subtraction.[6]

Kepler must have written back asking for a copy of the machine for himself because, on February 25, 1624, Schickard again wrote to Kepler giving a careful description of the use of the machine together with several drawings showing its construction. He also told Kepler that a second machine, which was being made for his use, had been accidentally destroyed when a fire leveled the house of a workman Schickard had hired to do the final construction.

Their two letters, both of which were found in Kepler's papers, give evidence that Schickard actually constructed such a machine. Unfortunately, the drawings of the machine had been lost and no one had the slightest idea of what the machine looked like or how it performed its arithmetic. Fortunately, some scholars, attempting to put together a complete collection of Kepler's works, were investigating the library of the Pulkovo Observatory near Leningrad. While searching through a copy of Kepler's Rudolphine Tables they found a slip of paper that had seemingly been used as a book mark. It was this slip of paper that contained Schickard's original drawings of the machine. One of these sketches is shown in Figure 1.18. Little detail can be seen, but with the hints given in the letters it became possible to reconstruct the machine.

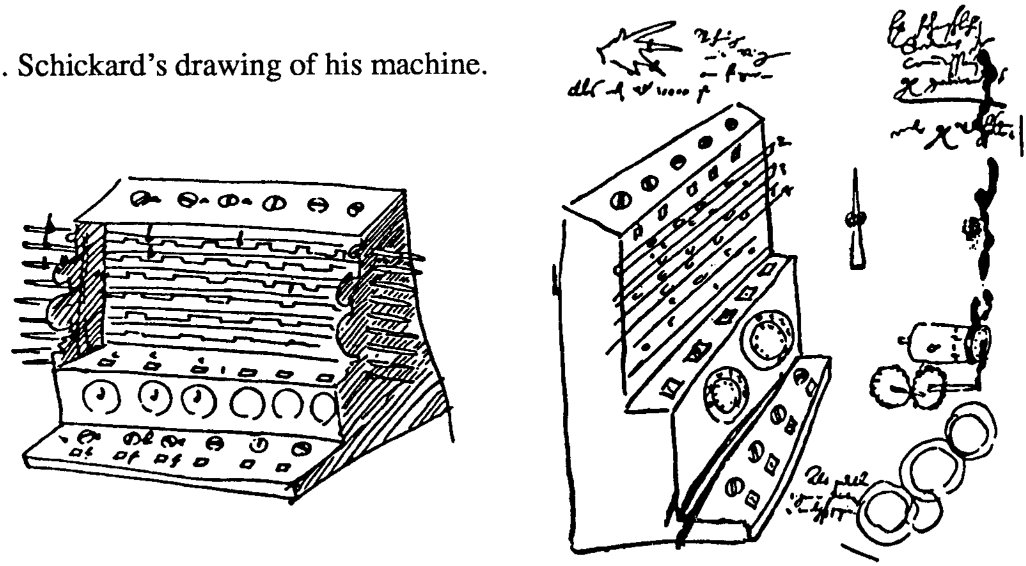


Figure 1.18. Schickard's drawing of his machine.

In the stamp illustration, the upper part of the machine is set to show the number 100722 being multiplied by 4. The result of this multiplication is added to the accumulator using the lower portion of the machine. The upper part is simply a set of Napier's bones (multiplication tables) drawn on cylinders in such a way that any particular "bone" may be selected by turning the small dials (marked *a* in Schickard's drawing). Moving the horizontal slides exposes

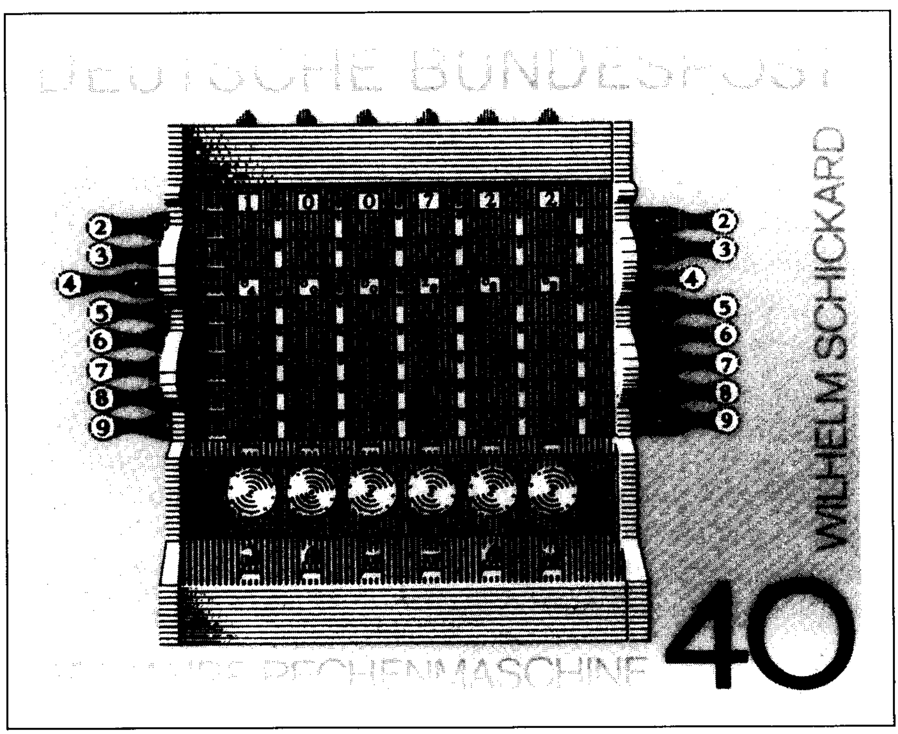
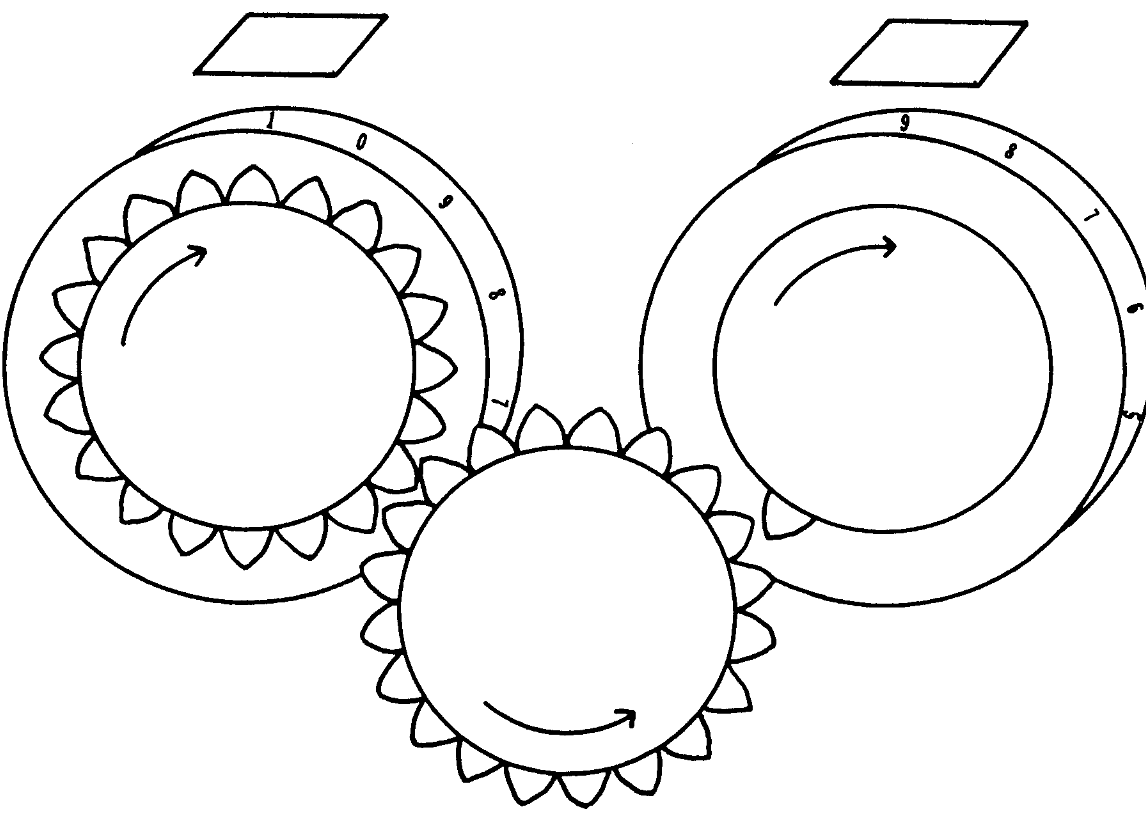


Figure 1.19. A stamp produced to honor the 350th anniversary of the invention of Schickard's machine.

different sections of the "bones" to show any single digit multiple of the selected number, the fourth multiple is shown exposed in Figure 1.19. This result can then be added to the accumulator by turning the large knobs (marked *d*) and the results appear in the small windows just above (marked *c*). The very bottom of the machine contains a simple *aide-memoire*. By turning the small knobs (*e*) it was possible to make any number appear through the little windows (*f*); this avoided the necessity of having pen, ink, and paper handy to note down any intermediate results for use at some later time in the computation.

The mechanism used to effect a carry from one digit to the next was very simple and reliable in operation. As shown in the drawing (Figure 1.20), every time an accumulator wheel rotated through a complete turn, a single tooth would catch in an intermediate wheel and cause the next highest digit in the accumulator to be increased by one. This simple-looking device presents problems to anyone attempting to construct an adding machine based on this principle. The major problem is created by the fact that the single tooth must enter into the teeth of the intermediate wheel, rotate it 36 degrees

Figure 1.20. The Schickard carry mechanism.



(one-tenth of a revolution), and exit from the teeth, all while only rotating 36 degrees itself. The most elementary solution to this problem consists of the intermediate wheel being, in effect, two different gears, one with long and one with short teeth, together with a spring loaded detente (much like the pointer used on the big wheel of the gambling game generally known as the "crown and anchor"), which would allow the gears to stop only in specific locations. It is not known if Schickard used this exact mechanism, but it certainly works well in the modern reproduction of his machine.

The major drawback of this type of carry mechanism is the fact that the force used to effect the carry must come from the single tooth meshing with the teeth of the intermediate wheel. If the user ever wished to do the addition 999,999 + 1, it would result in a carry being propagated right through each digit of the accumulator. This would require enough force that it might well do damage to the gears on the units digit. It appears that Schickard was aware of the limitations of the strengths of his materials because he constructed machines with only six digit accumulators even though he knew that Kepler would likely need more figures in his astronomical work. If the numbers became larger than six digits, he provided a set of brass rings that could be slipped over the fingers of the operators hand in order to remember how many times a carry had been propagated off the end of the accumulator. A small bell was rung each time such an "overflow" occurred, just to remind the operator to slip another ring on his finger.

Although we know that the machine being made for Kepler was destroyed in a fire, there is some mystery as to what happened to Schickard's own copy of the device. No trace of it can be found and it is unlikely to ever be found now that complete studies of Schickard's papers and artifacts have been done.

The Machines of Blaise Pascal (1623-1662)

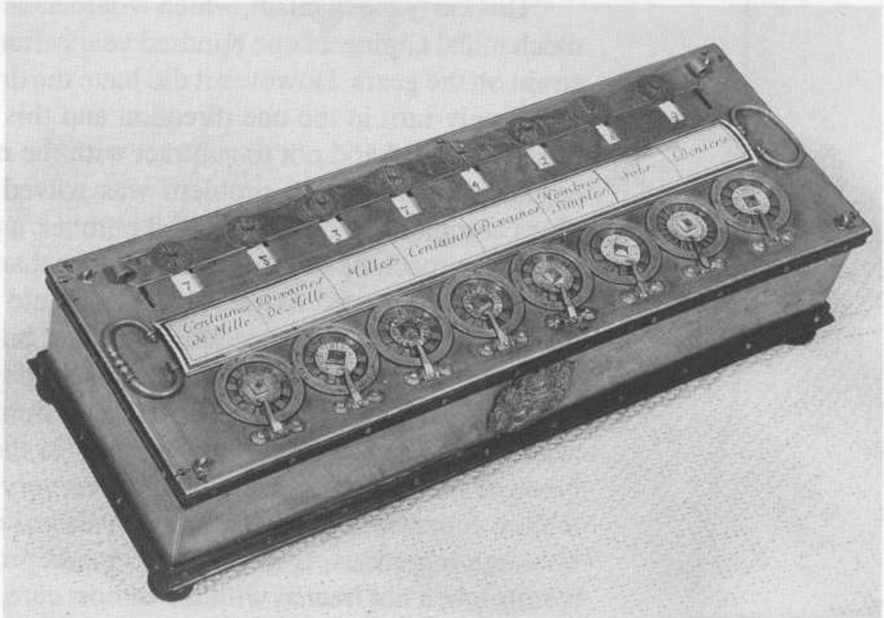
The great French mathematician and philosopher Blaise Pascal made the next major attempt to design and construct a calculating machine. The fact that he was not the first to construct such a device in no way reduces the magnitude of his achievement because his machine was entirely different from Schickard's and it is almost certain that Pascal would not have known of Schickard's machine, much less have seen it in operation.

Figure 1.21. Blaise Pascal (1623 - 1662).

Pascal came from the area of Clermont in southern France west of Lyon. The Pascal family was one of the noble houses of the area. When he was only nineteen years old he managed to design the first of his many calculating machines. He hired a group of local workmen and, showing them his carefully done drawings, asked them if they could make the instrument. What they produced was quite unworkable because they were more used to constructing houses and farm machinery than they were delicate instruments. This led Blaise to train himself as a mechanic, even spending time at a blacksmith shop to learn the basics of constructing metal parts. He experimented with gears made out of ivory, wood, copper, and other materials in an attempt to find something that could stand the strain of being used in a machine of his design.

Although he produced about fifty different machines during his lifetime, they were all based on the idea incorporated in his first machine of 1642. The device was contained in a box that was small enough to fit easily on top of a desk or small table. The upper surface of the box, as can be seen in Figure 1.22, consisted of a number of toothed wheels above, which were a series of small windows to show the results. In order to add a number, say 3, to the result register, it was only necessary to insert a small stylus into the toothed wheel at the position marked *3* and rotate the wheel clockwise until the stylus encountered the fixed stop, much the same way that you would dial a telephone today. The windows through which the results were read actually consisted of two separate sections, with a brass slide to cover the section not in use at the moment. The upper window was used for normal addition and the lower window, which displayed the nines complement (5 is the nines complement of 4 because 9 - 4 = 5) of the number held in the result register, was used for subtraction. This arrangement was necessary in that, due to the internal construction of the machine, it was not possible to turn the dials backwards in order to do a subtraction; instead one added the nines complement of the number one wished to subtract.

Figure 1.22. The top of Pascal's machine.



Pascal seems to have realized early on that the single tooth gear, like that used by Schickard, would not do for a general carry mechanism. The single tooth gear works fine if the carry is only going to be propagated a few places but, if the carry has to be propagated several places along the accumulator, the force needed to operate the machine would often be of such a magnitude that it would do damage to the delicate gear works. Pascal managed to devise a completely new mechanism that took its motive force from falling weights rather than from a long chain of gears.

The entire mechanism is quite complex, but the essentials can be seen in Figure 1.23. If the wheel marked *A* was connected to the units digit of the accumulator and the one marked *B* was connected to the tens digit, then any carry would be propagated from one to the other by the device marked *W* between the two shafts. Device *W* is a weight that is lifted up by the two pins attached to the wheel *A* as it rotates. When the wheel rotates from 9 to 0, the pins slip out of the weight allowing it to fall and, in the process, the little spring-loaded foot, shown in black, will kick at the pins sticking out of wheel *B*, driving it around one place. This gravity assisted carry mechanism was placed between each pair of digits in the accumulator and, when a carry was generated through several digits, could be heard to go "clunk," "clunk," "clunk" for each successive carry.

This carry mechanism, which would have been the pride of many mechanical engineers one hundred years after Pascal, eliminated any strain on the gears. However it did have the drawback that the wheels could only turn in the one direction and this meant that it was only possible to add and not to subtract with the machine. As mentioned earlier, the subtraction problem was solved by simply adding the nines complement of the required number, a process that limited the use of the machine to those with a better than average education.

Pascal attempted to put the machine into production for his own profit. This was not a successful venture, but it did result in a large number of units surviving to the present day. They are all slightly different in that they have different numbers of digits in the accumulator or have slight differences in the internal mechanisms. None of the surviving models functions very well, and it is doubtful if they functioned perfectly even in Pascal's day. The mechanism, although ingenious, is rather delicate and prone to giving erroneous results when not treated with the utmost care. Some of them will, for example, generate extra carrys in certain digits of the accumulator when they are bumped or knocked even slightly.

The Machines of Gottfried Wilhelm Leibniz (1646-1716)

Gottfried Wilhelm Leibniz was born in Leipzig on July 1, 1646. His father, a professor of moral philosophy, only lived until Leibniz was six years old, but he and his library were a great influence on the young Leibniz's early education. After he obtained a doctor of laws degree, the University of Altdorf offered him a professorship.

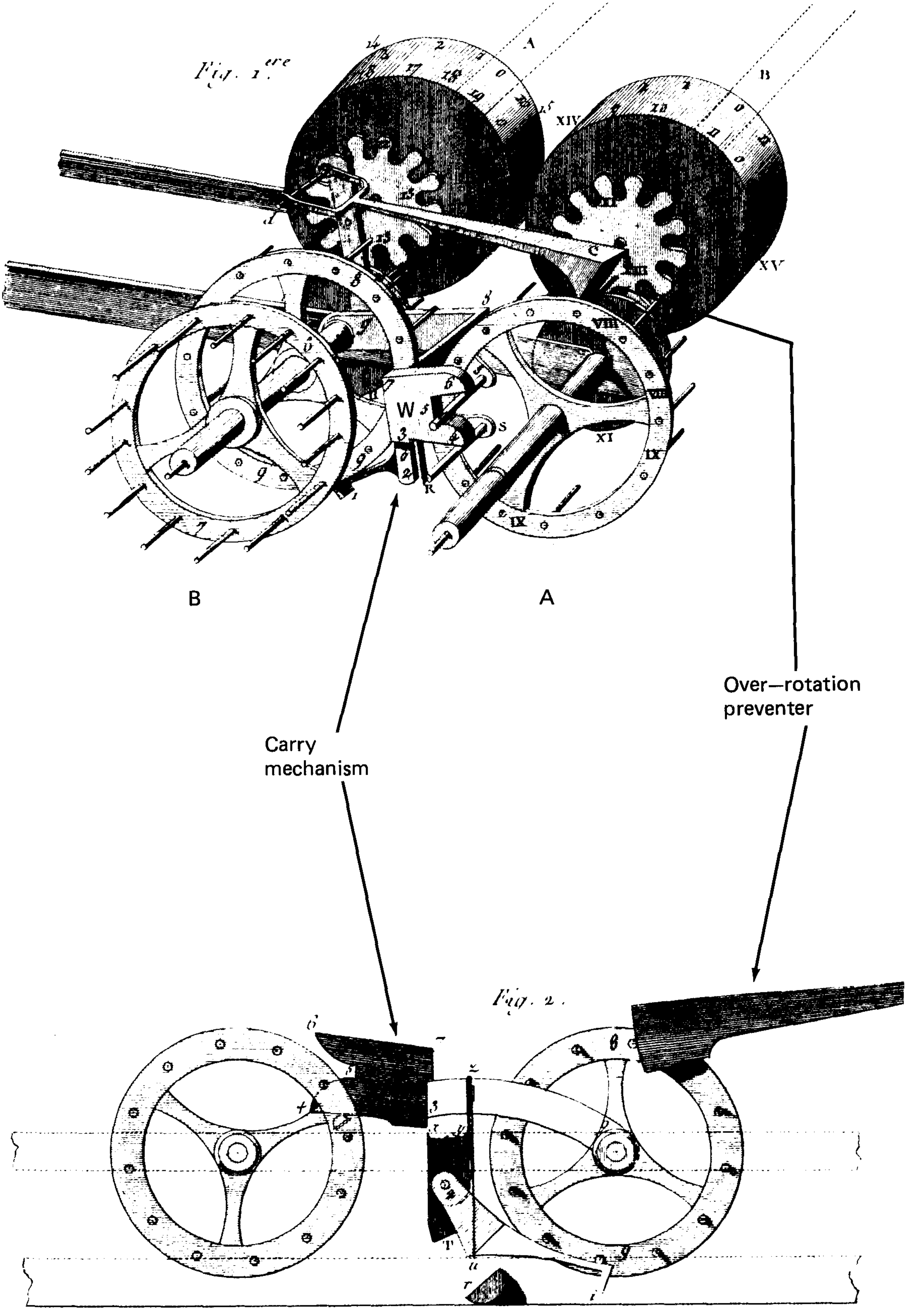


Figure 1.23. The internal workings of Pascal's machine, including the carry mechanism.

Figure 1.24. Gottfried Wilhelm Leibniz (1646-1716).

Wanting a more active job, he refused the offer and accepted a job as an advisor to the Elector of Mainz, one of the most famous statesmen of his day.

While he was in service to the Elector of Mainz he traveled a great deal to other European countries, acting as the elector's personal representative. During these travels he managed to meet most of the famous men of his day. This resulted in his being made a member of the British Royal Society and, later, a member of the French Academy.

Exactly when Leibniz became interested in the problem of mechanical calculation is not certain. It is known that when he heard that Pascal had invented a mechanical adding machine he wrote to a friend in Paris asking for details of its construction. We do not know if Leibniz ever actually saw one of Pascal's machines, but we do know that, at least in his early years, he did not completely understand its workings. In Leibniz's notes is a series of suggestions and drawings for an attachment to be placed on top of Pascal's device in order to enable it to perform multiplication. Although it was an interesting idea, the device could not have worked because no more than one wheel of Pascal's machine could rotate at any given instant. Presumably Leibniz either found this out or the pressure of other work caused him to put the idea aside until it no longer had any relevance, for he never seems to have continued along this line of thought.

The machine for which Leibniz is most famous, his mechanical multiplier, was actually lost to us for about two hundred years. Many records exist to prove that he had actually constructed a machine, but the actual device was not known. It appears that sometime in the late 1670s the machine was given to A. G. Kastner at Göttingen for overhauling and that somehow it was stored in the attic of one of the buildings of Göttingen University, where it remained for the next two hundred years. In 1879 a work crew attempting to repair a leaking roof discovered it lying in a corner. The workings of the machine are based upon one of Leibniz's inventions, the stepped drum, as illustrated in Figure 1.25.

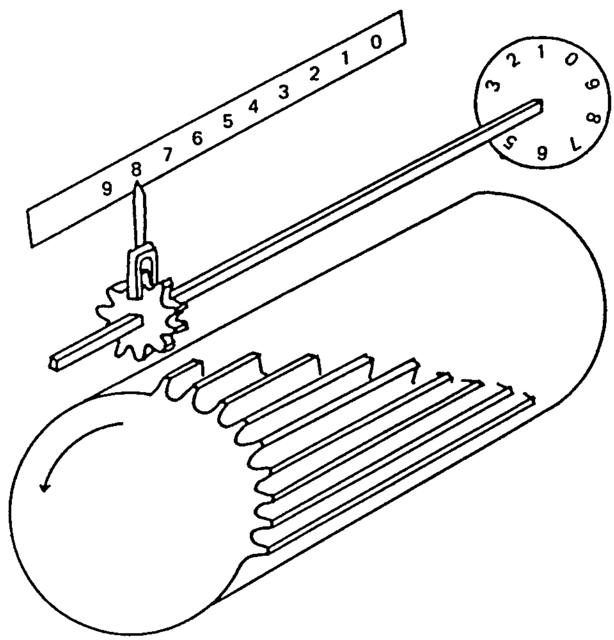


Figure 1.25. The Leibniz stepped drum mechanism.

A result wheel, shown at the end of the square shaft, could be rotated to any of ten different positions to register the digits 0 to 9. In order to add a quantity, say 8, to the result indicated on the wheel, it was only necessary to cause the square shaft to rotate 8 steps. This was done by having the small gear on the shaft mesh with 8 teeth on the large drum below the shaft. The small gear could slide up and down the square shaft so that, depending on its position, it would interact with a different number of teeth on the major drum. Leibniz's machine had eight of these mechanisms so that, when a number was registered on the machine by setting the small pointers (which controlled the position of the gears on the square shafts), a turn of a crank would cause all eight stepped drums to rotate and add the digits to the appropriate counters. To multiply a number by 5, one simply turned the crank five times. The actual machine was constructed in two layers so that, when one needed to multiply a number by 35 the following steps were performed:

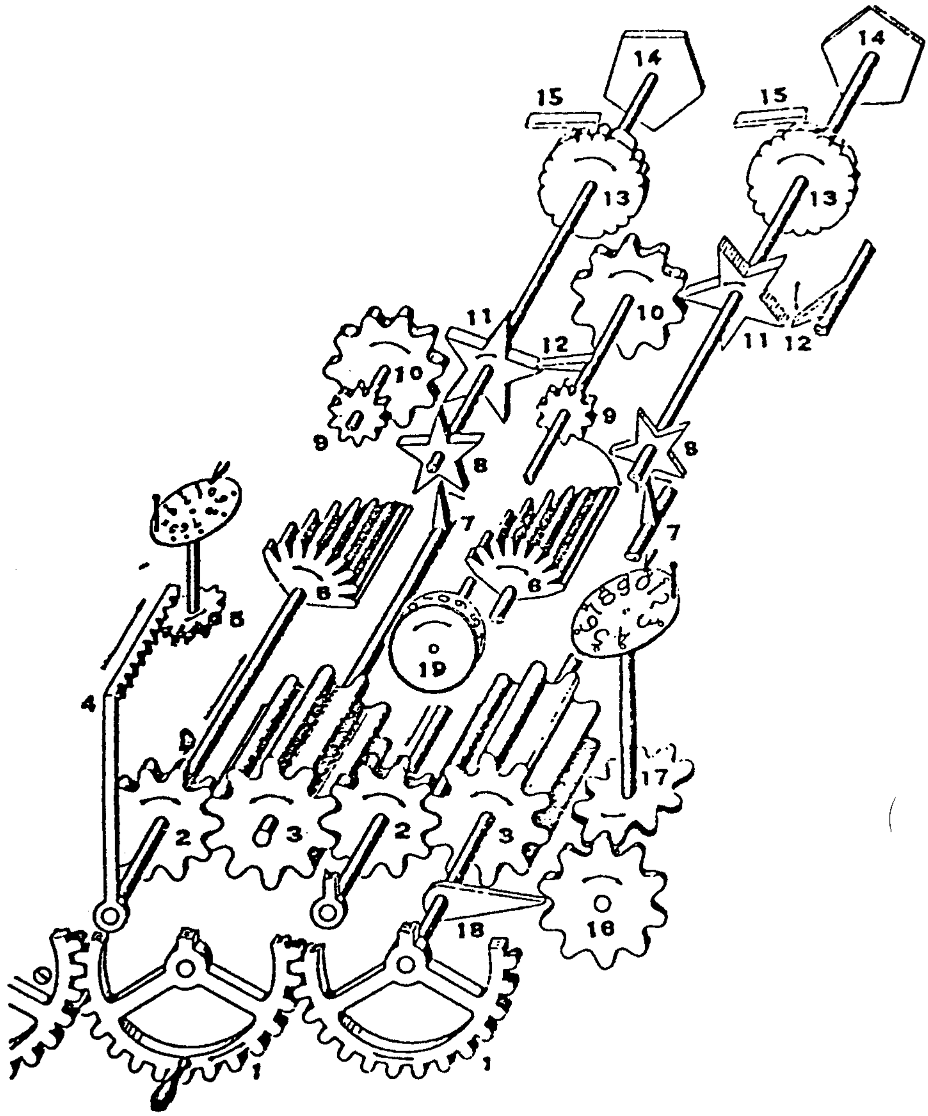
1. the number to be multiplied was set up by moving the gears along the square shafts so that the pointers indicated the desired number;
2. the crank was turned five times;
3. the top layer of the machine was shifted one decimal place to the left; and
4. the crank turned another three times.

One of the biggest problems when attempting to design this type of machine is how to deal with the possibility of a "carry" being generated from one digit to the next when the first digit rotates from the 9 position through to the 0 position. Leibniz only partially solved this problem. Although it appears complex, the diagram of the full mechanism is really quite simple when explained. Figure 1.26 shows two digit positions of the machine, the stepped drums being denoted by the digit *6*. The gears in front (labeled *1*, *2*, and *3*) are really just part of the drive mechanism and can be ignored. The more complicated mechanics, consisting of the levers, star wheels, cogs, and pentagonal disks (*12*, *11*, *10* and *14*) are all part of the carry mechanism.

When a carry was needed, the small lever *7* would interact with the star wheel *8* and partially turn the shaft so that one of the points of the star *11* would assume a horizontal position (compare the two star wheels marked *11* to note the two different positions they could assume). This would put it into a position in which the lever *12* (which turns once for each turn of the addition crank) could give it a little extra push to cause the result wheel to flip over to the next digit (i.e., add the carry to the next digit).

Note that this does not complete all the requirements of the carry

Figure 1.26. The full mechanism of the Leibniz machine.



mechanism, for this carry could, in turn, cause another carry in the next higher digit. There is no way that this simple mechanism can be used to ripple a carry across several digits. Note the two different positions of the pentagonal disk *14*: it can have a flat surface uppermost (which would be flush with the top cover of the machine and, thus, not noticeable to the operator) or it could have one of its points projecting above the top surface. This disk is so arranged that whenever a carry is pending, the point is up and when the carry has actually been added into the next digit, the point is down. After turning the crank to add a number into the register, if a ripple carry was generated, one or more of these points would project from the top of the machine, indicating that the need for a carry was detected but that it had not yet been added to the appropriate digit. The operator could reach over and give the pentagonal disk a push to cause the carry to be registered on the next digit and the point to slide back down into the mechanism. If that carry, in turn, caused another carry, further pentagonal disks would push their points through the slots in the top of the machine to warn the operator that he had to give the machine a further assist.

We know that Leibniz started to think about the problems involved in designing such a machine sometime about 1671. In January of 1672 he happened to be in London and was able to demonstrate a wooden model (which did not work properly) to the members of the Royal Society. Leibniz promised to make some technical changes and bring his machine back when it was properly functional. The secretary of the Royal Society did not invite Leibniz to the next meeting but suggested that when a proper working model was available they would like to have it demonstrated. Several letters remain in existence between the secretary and Leibniz concerning the progress of the machine over the next two years.

But Leibniz was plagued by the same types of problems that were faced by Pascal and others—poor workmen and poor materials with which to work. The final machine was only put together because Leibniz had found, during his stay in Paris, a French clockmaker named Olivier, who was both honest and a fine craftsman. No one knows for sure, but it is assumed that Leibniz simply explained the problems to M. Olivier and then let the clockmaker get on with the real construction work. The final version of the machine, which is now housed in the Landesbibliothek in Hannover, was put together in the summer of 1674.

As previously mentioned, the machine consists of two basic sections, the upper one contains the setup mechanism and the result register; the lower part, the basic Leibniz stepped gear mechanism. When the multiplicand digits have been entered into the setup slides, the handle on the front is turned once for every time that the multiplicand should be added to the answer dials. The large dial on the top right of the machine has a pin to set into it at the position

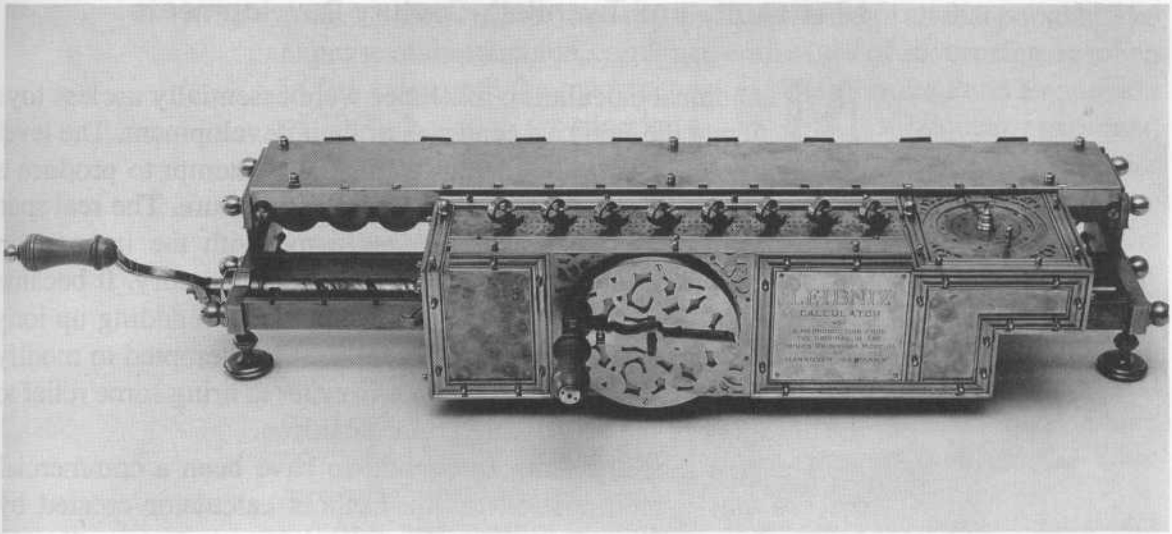


Figure 1.27. The Leibniz calculator.

indicated by the multiplier digit (e.g., 5) and, after five turns of the front handle, it brings this pin up against the stop to be seen at the top of the dial, preventing the operator from adding the multiplicand to the result too many times. After a single digit of the multiplier is processed, the crank at the far left of the machine is turned once to shift the top section of the machine over by one digit place so that the next digit of the multiplier can be considered. Thus, this machine was simply the mechanical version of the common shift-and-add procedure used for multiplication on many digital computers.

Leibniz is more widely known for his work in mathematics and philosophy than for his invention of a calculating machine. It is interesting to note, however, that the principle of the stepped drum gear was the only practical solution to the problems involved in constructing calculating machines until late in the nineteenth century.

Leibniz died on November 14, 1716, enfeebled by disease, harassed by controversy (not the least with Newton over the invention of calculus), and embittered by neglect. Men like him are often very difficult to get along with and there was an almost audible sigh of relief from his contemporaries when he finally died. An eyewitness tells us that

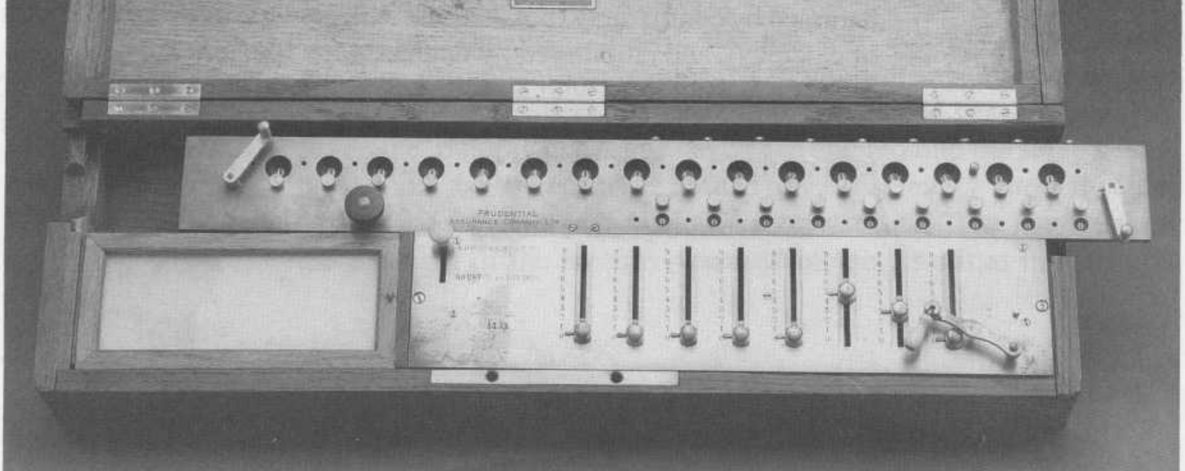
> he was buried more like a robber than what he really was, the ornament of his country.[7]

Nineteenth- and Twentieth-Century Developments

Mechanical calculating machines were essentially useless toys during the first two centuries of their development. The level of technology of the day guaranteed that any attempt to produce a reliable, easy to use instrument was doomed to failure. The real spur to the production of sound machines came with the increase in commercial transactions in the early nineteenth century. It became quite obvious that many hours were being spent in adding up long columns of figures, and many different people attempted to modify the older designs and create new ones in order to bring some relief to the drudgery of the accounting house practices.

The first machine that can be said to have been a commercial success was a modification of the Leibniz calculator created by Charles Xavier Thomas de Colmar, a French insurance executive, in 1820. Although Thomas was not aware of the early work of Leibniz, the internal workings of the machine rely on the same stepped drum principle. Thomas was able to produce an efficient carry mechanism and, in general, the machine was very well-engineered for its day (Figure 1.28). The Thomas firm developed many different models of the basic system and it remained in production until the start of the twentieth century. Although it had been available to the general public

Figure 1.28. An early example of the Thomas de Colmar Arithmometer. Rick Vargas photograph; courtesy Smithsonian Institution.



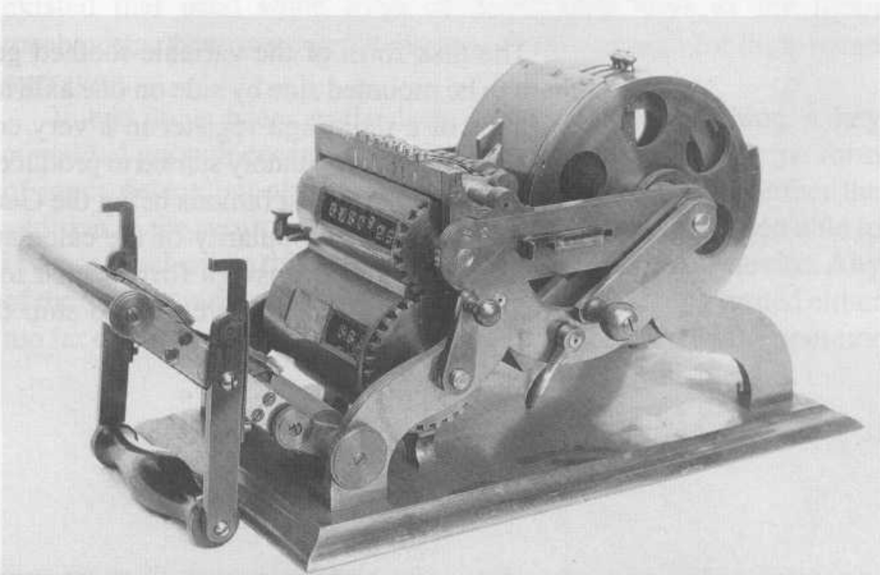
from the early 1820s, the early versions were not all that popular. The expense of the machine, combined with a lack of advertising, resulted in few sales until the machine was exhibited in the Paris Exposition of 1867. It was so far superior to the one other calculator exhibited that it won praise from the judges and finally became quite popular for both business and scientific calculations.

Like any good idea, the Thomas Arithmometer resulted in the production of many rival machines. Several different arrangements of the Leibniz stepped drums were tried, both to avoid simply having a carbon copy of the Arithmometer and in an attempt to reduce the size and weight of the resulting device. One of the most successful of these was the Edmonds Circular Scientific Calculator, which arranged the drums and associated gearing in a circle, the drive mechanism being a crank protruding from the top of the box.

Any real attempt at creating a smaller mechanical calculating machine had to wait until some mechanism was developed that could replace the Leibniz drum with a smaller and lighter device. The purpose of the drum was to provide a mechanism for engaging a gear with a variable number of teeth and, until late in the 1800s, no one had managed to find a workable system to produce gears that could quickly change the number of teeth projecting from their surface.

The true variable-toothed gear appeared in both Europe and America at about the same time. In America Frank S. Baldwin managed in 1873 to construct a model of a calculating machine, based on his invention of a variable-toothed gear. He immediately applied for a patent on the idea that, when granted in 1875, resulted in the device becoming known as "Baldwin's 1875 machine" (Figure 1.29).

Figure 1.29. The Baldwin 1875 machine. Courtesy Smithsonian Institution.



It was only three years later when Willgodt T. Odhner, a Swede working in Russia, produced almost the exact system in Europe. This coincidence resulted in this type of machine being referred to as a Baldwin machine in America and an Odhner machine in Europe. Odhner never claimed to have invented this style of machine and, in his first American patent, he makes it quite clear that he limits his claims to simply making several improvements in the design.

The concept of the variable-toothed gear is quite simple, as can be seen in Figure 1.30. A cam mechanism can be rotated by means of a lever so that as the cam contacts the different spring loaded rods they are forced to protrude from the surface of the disk in which they are mounted. Thus, it is possible to set the lever to the fifth position, resulting in a gear having five teeth. When this gear is rotated, the five teeth cause a result wheel to be turned to indicate that the number 5 has been added to whatever digit had been stored on the wheel.

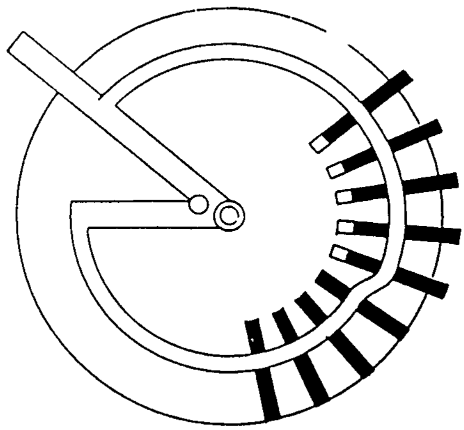


Figure 1.30. The variable-toothed gear mechanism.

The disk form of the variable-toothed gear allows a number of them to be mounted side by side on one axle to provide the arithmetic facilities of a multidigit register in a very compact package. Many different firms immediately started to produce machines based on this design, one of the most famous being the German firm of Brunsviga (Figure 1.31). The popularity of the calculator can be judged from the records of the Brunsviga firm, which indicate that they started production in 1892 and were able to ship their twenty thousandth machine in 1912.

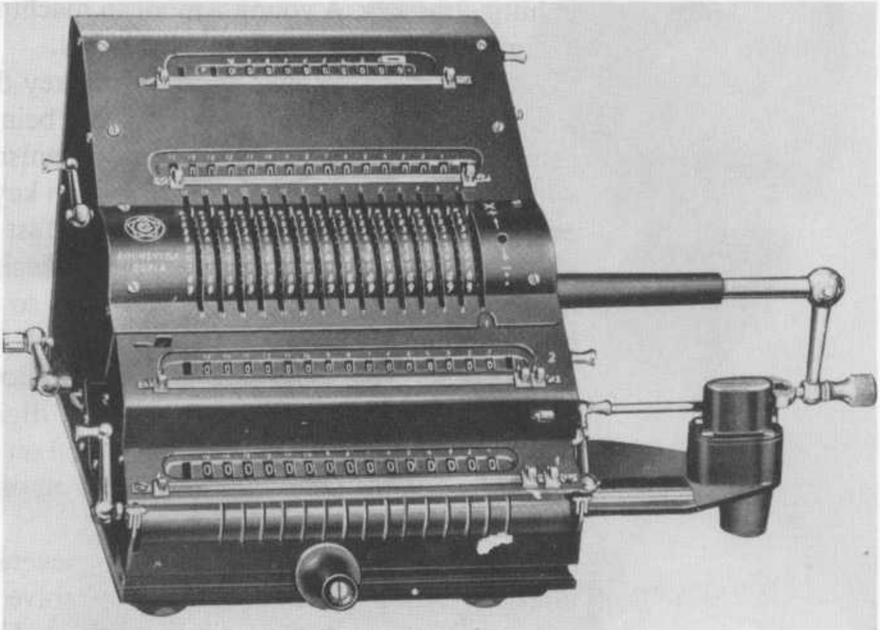


Figure 1.31. A Brunsviga calculating machine (Dupla model). The levers for setting the variable-toothed gears are in the central portion of the device.

All of these machines were better suited to scientific calculations requiring many operations on a few numbers than they were to the problem of adding up long lists of numbers often found in business applications. The labor of setting up a number on the machine, by moving a slide on the Arithmometer type of machine or setting a lever on the Brunsviga type, was slow enough that it made the devices impractical for many commercial firms. Although various models existed that used some form of depressible keys as the input mechanism, these were generally not reliable enough for high-speed operation.

It had long been realized that the action of pushing a key contained enough energy not only to set the number on some form of input device but also to cause the gears to rotate and effect the addition to the result wheels. Unfortunately, no one had been able to invent a mechanism that incorporated both actions in one device. Any of the early attempts usually had the result wheels being turned either too far or not far enough, depending on the force used by the operator in hitting the key. A young American machinist, Dorr E. Felt, found a workable solution in the middle 1880s.

All the early attempts at producing key-driven adding machines relied on the action of depressing a key being communicated to the result wheel by means of a ratchet mechanism that rotated the result wheel by an amount dependent on which key had been pushed. Not only was it found impossible to stop the fast moving result wheel in the proper location but any mechanism designed to carry a digit to the next higher result wheel was always so slow in its action as to limit drastically the speed of operation. A highly trained operator could push keys at a rate that would only allow 1/165 of a second for any carry to be transmitted to the next digit. This meant that any attempt at producing a mechanism based on something as simple as the odometer system found in modern automobiles was doomed to failure.

Dorr E. Felt managed to invent several different mechanical arrangements that he thought might solve most of the problems inherent in a key-driven adding machine. Unable to afford to have his ideas properly constructed from metal, he built his first prototype from rubber bands, meat skewers, staples, bits of wire, and an old macaroni box for the casing (Figure 1.32).

Felt set up a partnership with a man named Robert Tarrant in 1887 and the pair of them started producing commercial quantities of "Comptometers." The success of their key-driven model (Figure 1.33) was so spectacular that no other key-driven adding machine was able to compete with it until after 1912.

One of the next major advances in the production of calculating machines was the incorporation of special devices to automate the operations of multiplication and division. These developments actually took place simultaneously with the Baldwin and Odhner inventions, but they were generally incorporated into machines based on the older Thomas de Colmar design. In all earlier machines it was necessary to perform multiplication by a series of repeated addition operations. This usually required the operator to turn the machine's crank as many times as was represented by the sum of the digits of the multiplier. Single-digit, or even two-digit, multipliers presented little problem when working with the Thomas type of machine, but multipliers of many digits resulted in both the expenditure of considerable physical effort and the passing of long periods of time before the answer could be obtained.

The usual mode of operation in a machine with automatic

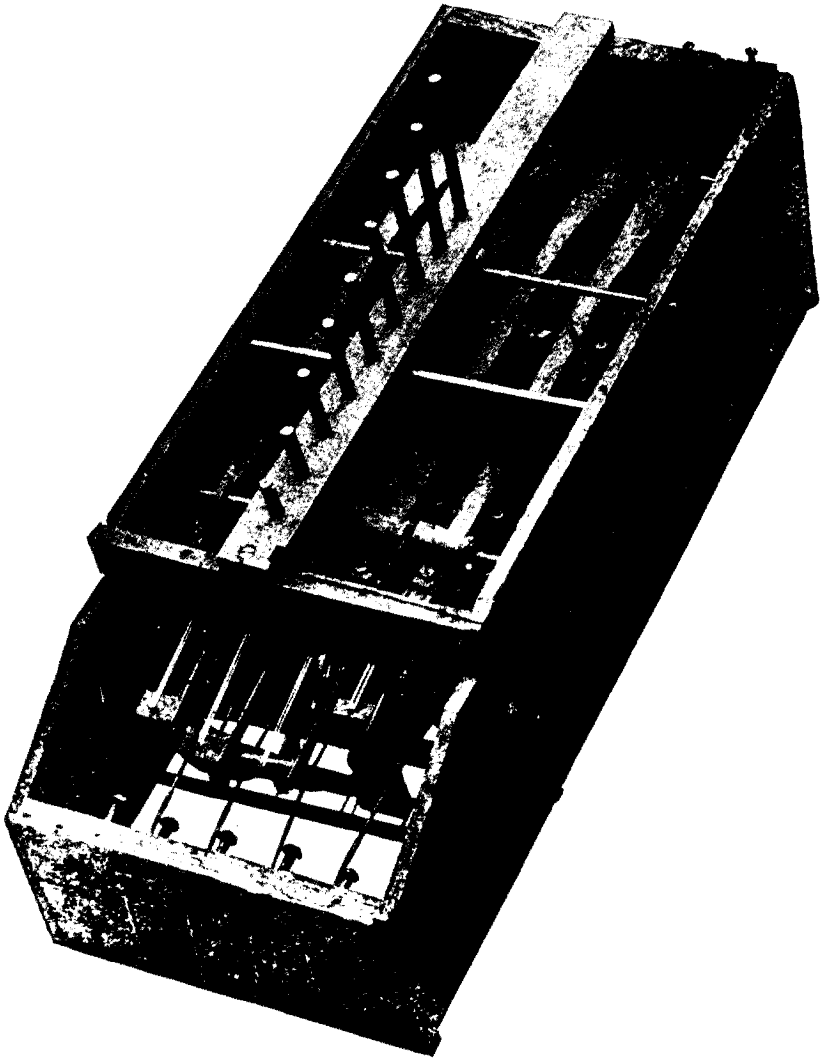
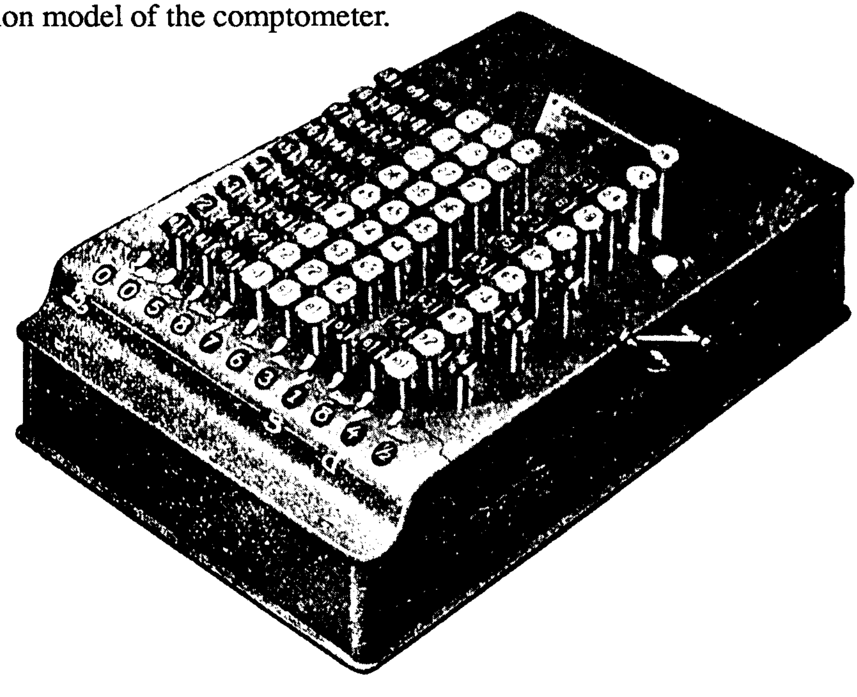


Figure 1.32. Felt's macaroni box model.

Figure 1.33. A production model of the comptometer.



multiplication features required that the handle be turned only once for each digit in the multiplier. Typical, and perhaps most popular, of these automatically multiplying machines was the "Millionaire" (Figure 1.34) invented by Otto Steiger of Munich in the early 1890s. Steiger started manufacturing the Millionaire in Zurich and, because of its speed and reliability, it was soon being sold to scientific establishments throughout Europe and America. Its popularity lasted until 1914, when the First World War interrupted the organization of sales and support.

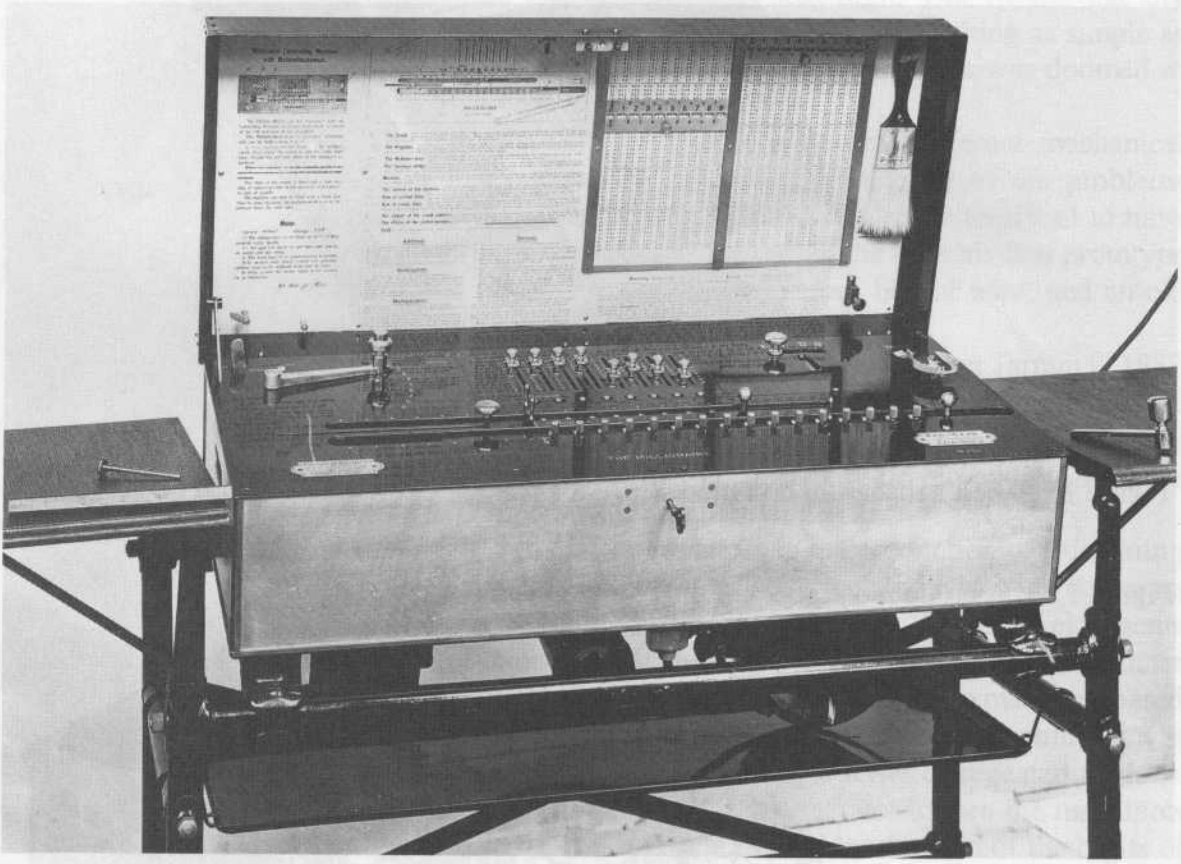


Figure 1.34. A Millionaire. Courtesy Science Museum.

The late nineteenth and early twentieth centuries saw many other firms start to produce calculating machines of different types. By the start of the First World War it was possible to obtain easily machines that incorporated automatic mechanical multiplication devices (much like a mechanical version of Napier's Bones), machines that could print their results on paper or ledger cards, machines that were driven by both electric or spring-driven motors, and even machines having a combination of these features. Several specialty firms even produced machines that consisted of many calculators ganged together in different ways in order to simplify certain special types of calculations. Once the basic technology had been developed, only the limit of human imagination (and the laws of physics) constrained the different forms taken by mechanical calculators. They ranged from desk-sized objects full of features to small examples that were based on Swiss watch technology and capable of being held in one hand yet able to perform all the basic arithmetical functions.

Notes

1. H. Wassen as quoted by James R. Newman, in *The World of Mathematics* (New York: Simon and Schuster, 1956), 463-464.

2. In D. E. Smith, *History of Mathematics, Vol. II* (New York: Dover Publications, 1958), 516.

3. In C. G. Knott, ed., *Napier Tercentenary Memorial Volume* (London: Longmans, Green for the Royal Society of Edinburgh, 1915), 126.

4. Mark Napier, *Memoirs of John Napier* (Edinburgh: William Blackwood, 1834), 410.

5. In R. T. Gunter, *Historic Instruments for the Advancement of Science* (Oxford: Oxford University Press, 1925), 25.

6. As quoted in a lecture at Los Alamos by Dr. Baron von Freytag Loringhoff, 1975.

7. *Encyclopaedia Britannica*, 11th ed., s.v. "Leibniz."

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